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# Investment in Two Sided Markets and the Net Neutrality Debate

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This paper develops a game theoretic model based on a two-sided market framework to investigate net neutrality from a pricing perspective. In particular, we consider investment incentives of Internet Service Providers (ISPs) under both a neutral and non-neutral network regimes. In our model, two interconnected ISPs compete over quality and prices for heterogenous Content Providers (CPs) and heterogeneous consumers. In the neutral regime, connecting to a single ISP allows a CP to gain access to all consumers. Instead, in the non-neutral regime, a CP must pay access fees to each ISP separately to get access to its consumers. Hence, in the non-neutral regime, an ISP has a monopoly over the access to its consumer base. Our results show that ISPs' quality-investment levels are driven by the trade-off they make between softening price competition on the consumer side and increasing revenues extracted from CPs. Specifically, in the non-neutral regime, because it is easier to extract surplus through appropriate CP pricing, ISPs' investment levels are larger. Because CPs' quality is enhanced by ISPs' quality, larger investment levels imply that CPs' profits increase. Similarly, consumer surplus increases as well. Overall, under the assumptions of our model, social welfare is larger in the non-neutral regime. Our results highlight important mechanisms related to ISPs' investments that play a key role in market outcomes, providing useful insights for the net neutrality debate.

*Key words*: Two Sided Markets, Net Neutrality, Investments

## 1. Introduction

Since 2005, when the Federal Communications Commission (FCC) changed the classification of Internet transmissions from "telecommunication services" to "information services," Internet Service Providers (ISPs) are no longer bound by the non-discrimination policies in place for the telecommunications industry (Federal Communications Commission 2005). This has led to the so called net neutrality debate. While there is no standard definition of what a net neutral policy is, it is widely viewed as a policy that mandates ISPs to provide open-access, preventing them from any form of discrimination against Content Providers (CPs). We study one form of discrimination that could arise when ISPs charge CPs that are not directly connected to them for access to their consumer base. Even though there is no legislation that enforces this, under current practice, ISPs charge only CPs who are directly connected to them. Looking at net neutrality from a pricing perspective raises the question of what limits, if any, should be placed on pricing policies of ISPs. More explicitly, should an ISP be allowed to charge off-network CPs—those that are not directly connected to the ISP—who want to deliver content to its consumers or should the status quo remain?

Net neutrality has been a widely and hotly debated issue by law and policy makers. On one side of the debate are CPs who fear that if ISPs are allowed to charge off-network CPs, ISPs will engage in practices that will threaten innovation. Specifically, they argue that the flexibility in network pricing that the lack of legislation allows will be misused by ISPs to charge inflated prices, since they would have a monopoly over the access to their consumer base. In short, the high prices would deter entry, reduce CP surplus and CPs' innovation incentives, especially affecting nascent CPs. The other side of the debate is advanced by ISPs who argue that net neutrality regulation would hinder their ability to recoup investment costs on their broadband networks, essentially taking away the economic incentives to upgrade their infrastructure.[1]

The above debate has mostly been of a qualitative nature (see, e.g., Wu 2003, Sidak 2006, Yoo 2006, Hahn and Wallsten 2006, Faulhaber 2007, Frieden 2008, Lee and Wu 2009); with some notable exceptions notwithstanding (see, e.g., Economides and Tåg 2007, Choi and Kim 2008, Musacchio et al. 2009), not much formal economic analysis has been done to shed light on the validity, or lack thereof, of these arguments. Our research adds to the growing body of formal economic analysis that will help inform policy makers on the net neutrality debate. In particular, this article develops a game theoretic model based on a two-sided market framework (for an introduction to two-sided markets, we refer the reader to Rochet and Tirole 2006) to investigate net neutrality as a pricing rule; i.e., whether there should be a mandate to preserve the current pricing structure. To understand the effects of such a policy on the Internet, we study its effect on investment incentives of ISPs and its concomitant effects on social welfare, consumer and CP surplus, and CP market participation. Our work complements, and in some cases challenges current literature on net neutrality, providing useful insights for this policy debate.

Our model consists of two interconnected ISPs represented as profit maximizing platforms that choose quality investment levels and then compete in prices for both CPs and consumers. There is a mass of CPs that are heterogenous in content quality, and a mass of consumers that have

[1] This argument is perhaps best exemplified by the former CEO of AT&T, Ed Whitacre, who said in an interview that *"Now what [CPs] would like to do is use my pipes free, but I ain't going to let them do that because we have spent this capital and we have to have a return on it. So there's going to have to be some mechanism for these people who use these pipes to pay for the portion they're using"* (Business Week 2005).

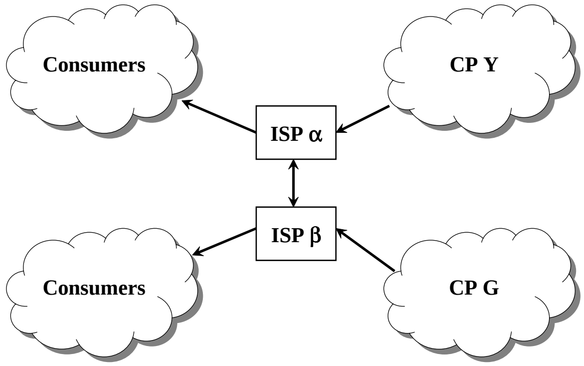


**Figure 1** In a neutral model, CP $G$ only pays to platform $\beta$ to get access to all consumers. In a non-neutral model, $G$ needs to pay to both platforms, otherwise it cannot get access to consumers connected to ISP $\alpha$.

heterogenous tastes over content. Platforms provide connection services to consumers and CPs and charge a flat access fee to both. A CP makes revenue from advertising, which is increasing with the mass of consumers that access it, as well as with the quality of its content that is enhanced by the quality of the connections between the CP and consumers. Based on this, CPs make connection decisions; the mass of CPs that decide to participate in the market serve as a proxy for CP innovation in our model. Consumers gain value from the content provided by CPs. A consumer's utility is increasing in the mass of CPs it has access to, the quality of these CPs, and the quality of the connection between the consumer and the CPs. To incorporate congestion in the model, the quality of a consumer-CP connection is given by the bottleneck (i.e., worst) quality between both platforms involved in the connection.[2]

Our analysis involves a neutral and a non-neutral model. The difference between the two models is the pricing structure employed. In a neutral model a CP pays only once to access the Internet, and through its ISP it can communicate with consumers subscribed to either platform. Instead, in a non-neutral model a CP pays additional fees to reach off-network consumers. To illustrate with the example in Figure 1, in a neutral regime, if a CP $G$ (e.g., Google) pays ISP $\beta$ (e.g., Comcast) to connect to it, $G$ has access to all consumers regardless which platform the consumers are connected to. In contrast, under a non-neutral regime, ISP $\alpha$ will allow CP $G$, who is not in its network, to reach its subscribers only if $G$ makes payments to $\alpha$. In that sense, in the non-neutral regime, each platform has a monopoly over the access to its consumer base.

[2] We highlight that our model abstracts away many features of the topology of Internet. We do so mainly for tractability reasons. The real structure of the Internet is more complex and contains more entities grouped in intricate ways. There are hierarchies of ISPs who connect creating complex topologies and peering agreements. Also CPs place their content closer to users by using server farms or content distribution networks. For an overview of the current business structure, interconnections, agreements and contracts, we refer the reader to Crowcroft (2007) and Yoo (2010). Although we do not consider all those factors present in reality, we believe that our model captures important first-order effects of the relationships between competing ISPs, CPs and users.

In the neutral and the non-neutral regimes, we model the interaction between ISPs, CPs and consumers as a six-stage game that incorporates the different time-scales at which decisions are made. The timing is given by platforms' investment decisions (stage 1), CP competition (stages 2-3), and consumer competition (stages 4-6). Competition at each side of the market corresponds to a pricing game with vertical differentiation followed by a choice of platform for the agents on that side. Further details of the six stages are given in Section 2. Technically, these games are involved to solve because of the many stages and the heterogeneity among the participants. However, notably, we are able to explicitly solve for the subgame perfect equilibria of these games using backward induction.

We provide an explicit characterization of equilibrium investment levels, prices and market coverage levels under both the neutral and non-neutral regimes. We show how the outcome depends on the consumer mass $f$, and the distribution of CP quality, summarized by the average quality $\overline{\gamma}$ and a parameter that measures heterogeneity in quality $a$. Under the assumptions of our model, the first major result shows that platforms investment patterns are driven by trade-offs between softening price competition on the consumer side and increasing profits on the CP side. The result of the trade-off depends on whether the network is neutral or not. The next two bullet points provide details for each regime.

- In the neutral model the platforms are viewed as substitutes by both CPs and consumers. Hence at equilibrium, platforms maximally differentiate to corner different consumer and CP niches in the markets. More precisely, one platform opts not to invest while the other picks the highest quality permitted by investment costs. (Not investing is interpreted as investing the least possible amount to have an operating network.) In the sequel, we refer to the platform that invest the least, resp. the most, as the low-quality, resp. high-quality, platform. Essentially, the low-quality platform does not invest and trades-off making revenue on the CP side to making revenue on the consumer side. Investing does not pay off because it would increase price competition on the consumer side, thus reducing revenues extracted from consumers; this effect dominates the additional revenues that could be captured from CPs. In contrast, the investment made by the high-quality platform allows it to differentiate from the low-quality platform and to earn significant revenue from CPs as well as consumers. To put this in perspective, real-world ISPs indeed differentiate from each other by offering distinctive features such as various connection speeds and value-added services that enhance user experience like virus protection, spam filters, etc. (DiStefano 2008). In Section 4, we show the relationship between the investment level in the high-quality platform and the distribution of content quality among CPs.

- In the non-neutral model platforms are viewed as substitutes only by consumers. In contrast, from the CPs' perspective, each platform has a monopoly over the access to its consumer base.

Consequently, CPs decide whether to connect to each platform independently, causing different platforms' investment patterns from those in the neutral regime. Even though when the consumer base is large and the average CP quality is low platforms maximally differentiate for similar reasons to those alluded to in the neutral model, in all other cases platforms only differentiate partially. In particular, both platforms make positive investments leading to more investment in platform quality than in the neutral regime. In fact, in the non-neutral model, platforms can recoup their investments more easily through appropriate CP pricing. Consequently, the low-quality platform invests to increase revenues extracted from CPs; this effect is more important than softening price competition on the consumer side. Section 6 provides more details on the relation between the investment level of the low-quality platform and the distribution of quality among CPs.

Under the assumptions of our model, the non-neutral regime leads to a higher overall social welfare. This follows from the higher investment levels resulting in the non-neutral regime, which in turn increase consumer and CP gross surplus. (Gross surplus is defined as the total utility earned by a player before subtracting the price it pays to the platform.) Moreover, contrary to a popularly held opinion in the policy debate, under the assumptions of our model CPs' profits and consumer surplus are higher in the non-neutral regime. As before, this is driven by the additional investment made by the low-quality platform under the non-neutral regime. CPs increase their revenues from additional advertisement; this increase more than compensates for the larger price charged by the low-quality platform. Larger investment has two major effects on consumers. First, it increases price competition between platforms leading to lower connection prices. Second, it results in enhanced platform quality which translates into additional utility for consumers. Surprisingly, even though the low-quality platform prefers a non-neutral policy, the high-quality platform has the opposite preference. This is because under the non-neutral regime, the low-quality platform erodes the profits of the high-quality one by reducing differentiation; a neutral network involves maximal differentiation in quality. The table below summarizes the preferences of the network participants, indicated by check marks in the row corresponding to the preferred regime.

| Regime | CPs | Consumers | High-quality platform | Low-quality platform |
|---|---|---|---|---|
| Neutral | | | ✓ | |
| Non-Neutral | ✓ | ✓ | | ✓ |

Moreover, the difference in social welfare between the two regimes increases with the average CP quality and decreases with CP heterogeneity. An increase of the average CP quality heightens the incentive to invest for the low-quality platform, which causes a larger CP and consumer gross surplus. On the other hand, an increase in CP heterogeneity makes CP demand less elastic, leading to a lower incentive to invest for the low-quality platform. This, in turn, leads to lower CP and consumer gross surplus.

Our results suggest that investment incentives of ISPs, which are important drivers for innovation and deployment of new technologies, play a key role in the net neutrality debate. In the non-neutral regime, because it is easier to extract surplus through appropriate CP pricing, our model predicts that ISPs' investment levels are higher; this coincides with the predictions made by the defendants of this regime. Moreover, because CPs' quality is enhanced by platforms' quality, larger investment levels imply that CPs' profits increase. Similarly, consumer surplus increases as well. We note that the participation of CPs, our proxy for CP innovation, is not reduced in the non-neutral regime. Due to technical limitations, we did not include an investment stage for CP quality in our game. This may provide a more direct way of modeling CP innovation and may change some of our qualitative conclusions. However, we believe that the mechanisms related to ISPs' investments that our model highlights would also be present in this alternative model. Moreover, we believe that our results provide useful insights that can help policy makers make more informed decisions in this important policy debate.

The rest of this paper is organized as follows. In Section 2, we present the game that models the neutral regime. Section 3 characterizes a subgame perfect equilibrium of the game, and Section 4 discusses its properties and draws insights. In Section 5, we modify the game to model the non-neutral regime, while Section 6 presents our findings. In Section 7, we compare the resulting welfare in each regime. We conclude in Section 8 by summarizing our results and providing insight for policy makers. Due to space limitations all proofs have been relegated to the appendices.

### 1.1. Related Literature

As initially mentioned, much of the net neutrality debate has been qualitative; mostly from the law and policy sphere. In addition to the papers cited in the introduction, Farell and Weiser (2003), Nuechterlein and Weiser (2005), Chong (2007), Hogendorn (2007), Odlyzko (2009), Levinson (2009) also discuss various policy aspects of the net neutrality debate. Recently, a few publications have formalized some of the issues around net neutrality with mathematical models. Conceptually, it is useful to classify this emerging work into two broad classes categorized by the adopted working definition of net neutrality. One group views abandoning net neutrality as a licence to introduce differentiated service classes—or priority lanes—in the Internet (Hermalin and Katz 2007, Choi and Kim 2008, Schwartz et al. 2008, 2009, Krämer and Wiewiorra 2009, Cheng et al. 2010). In contrast, the other group views abandoning net neutrality as abolishing the current pricing structure in the Internet (Economides and Tåg 2007, Cañon 2009, Musacchio et al. 2009, Gupta et al. 2010).

The second group is more related to our work. Economides and Tåg (2007) use a two-sided market framework to investigate the effect of net neutrality regulation (defined as setting zero access fee to CPs) in both a monopoly and duopoly setting. They find that total welfare is higher in the neutral regime under both scenarios. However, their model does not include platforms investment decisions, a key driver of our results. Cañon (2009) investigates the effect of net neutrality under various pricing regulations in the presence of investment decisions. He finds that the neutral regime is superior in terms of total welfare. Unlike our setup, though, his model considers only a single monopolistic ISP, instead of looking at competitive platforms as we do. Gupta et al. (2010) analyze investment incentives of network providers under both congestion-based and flat pricing. Their results show that social benefits are generally higher in congestion-based pricing, illustrating the critical impact pricing structures have on network providers' investment incentives. Musacchio et al. (2009) is the closest to our work. They develop a two-sided market model and compare economic welfare across both regimes. The neutral one corresponds to "one-sided" pricing where only consumers are charged and the non-neutral one corresponds to "two-sided" pricing where both consumers and CPs are charged. According to this model, either regime can be superior with respect to overall welfare or that of CPs and ISPs. A detailed summary can be found in Schwartz and Weiser (2009). Although we use a similar definition of net neutrality, our model differs from theirs in a significant number of ways. In particular, the novel features of our model are:

- There is a continuum of CPs with heterogeneous quality, instead of homogenous and atomic. Advertising rates increase with CP and platform quality. In addition, the market coverage for CPs, our proxy for CP innovation, is endogenously derived. In contrast, they explicitly model CPs' quality investment decisions.
- There is a continuum of consumers that are heterogeneous in their tastes. Consumer connection decisions are endogenous and driven by competition between platforms; instead of assuming that consumers are split equally among ISPs and that ISPs are local monopolies.
- We model a bottleneck effect between platforms to highlight the impact of differentiated quality.

In our model, competition on the consumer side and the heterogeneity on both sides of the market are key drivers of the results. The lack of these elements in their model together with the other differences described above explain the dissemblance of our results from theirs.

On a broader scope, our work complements and contributes to previous research in the literature of Industrial Organization by explicitly considering quality investment in the context of two-sided markets. In particular, our model embeds price competition and quality choice in vertically differentiated markets (like in Shaked and Sutton 1982) in a two-sided market (like in Rochet and Tirole 2003, Gabszewicz and Wauthy 2004, Roson 2005, Parker and Alstyne 2005, Armstrong 2006). We note that investment decisions in two sided markets have received little attention in the literature. An exception is Farhi and Hagiu (2007) that considers investment as a strategic variable in a two-sided duopoly market model. However, their analysis investigates how investment strategies of an incumbent platform may help it to accommodate or deter entry of another platform. In our models, both platforms simultaneously compete in the investment stage.

In addition, in our model there are both negative (congestion) and positive externalities. In particular, consumers in one platform benefit from the presence of CPs in the other platform because of interconnection. This is in contrast to most two-sided market models in which both sides need to choose the same platform through which they do businesses (see Laffont et al. (2003) for an exception). We conclude by mentioning that Njoroge et al. (2009) present a preliminary version of the neutral model, which allowed us to build towards the models presented here.

## 2. The Neutral Model

We consider two platforms denoted by $\alpha$ and $\beta$, and a continuum of consumers with a mass of $f \in [0,1]$, and of CPs with a unit mass. Let $y_z \in \mathbb{R}_+$ be the quality-of-service (QoS) chosen by platform $z \in \{\alpha, \beta\}$. Without loss of generality, we assume that $y_\alpha \geq y_\beta \geq 0$, and hence, we refer to $\alpha$ as the high-quality platform and to $\beta$ as the low-quality one. Let $\gamma_j$ be the quality of CP $j$ where $j \in [0,1]$. Here, $\gamma_j$ is a uniformly distributed random variable with support $[\overline{\gamma} - a, \overline{\gamma} + a]$ and $0 < a < \overline{\gamma}$. We assume that the $\gamma_j$'s are independent and identically distributed across the population of CPs. Let $\phi : [0, f] \to \{\alpha, \beta\}$ and $\widehat{\phi} : [0,1] \to \{\alpha, \beta\}$ be the connection decisions that map the space of consumers and CPs, respectively, to the set of platforms. Aggregating those mappings, we denote by $r_\alpha$ and $r_\beta$ ($q_\alpha$ and $q_\beta$) the masses of CPs (consumers) that join each platform. Besides choosing their QoS as we will discuss below, platforms offer additional services to their own consumer base that improve the quality of content generated by CPs. Examples of such additional services are email accounts, virus scanning, blocking of malicious sites, and spam filtering. We denote the values of these services by $k_\alpha$ and $k_\beta$, defined as random variables with the same distributions as those of $\gamma_j$.

We now introduce the utilities of the three types of participants in this multistage game.

**Consumer Utility:** A consumer $i$ on a platform $\phi(i)$ connecting to a CP $j$ on platform $\widehat{\phi}(j)$ receives utility

$$u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, k_{\phi(i)}, r_{\widehat{\phi}(j)}) = \min\{y_{\phi(i)}, y_{\widehat{\phi}(j)}\} \left( \frac{\gamma_j}{r_{\widehat{\phi}(j)}} + k_{\phi(i)} \right). \tag{1}$$

The formula multiplies the quality of the network transmission, given by the worst of the two platforms, by the value of the content plus the additional services offered by the platform. To compute the value of the content, we divide the quality of CP $j$ by the mass of CPs that connect to the same platform to incorporate congestion effects: more CPs in a platform generate more congestion, reducing value. The value generated by both the CP and platform content is affected by the QoS of the transmission. This implies that a consumer on a high-quality platform, connecting to a CP on the same platform, receives higher utility than if he connects to a CP of the same quality on the low-quality platform. In essence, consumer utility depends on the platform that acts as a bottleneck, capturing common congestion effects present in the Internet (Akamai Technologies 2000).

Each consumer connects to a single platform but once connected has access to all content due to the interconnection of the platforms in the neutral model. In particular, a consumer $i$ on platform $\phi(i)$ connects to all CPs subscribed to either platform since $u_{ij} \geq 0$ for all $j$. We let the overall utility perceived by consumer $i$ that joins platform $\phi(i)$ be

$$F_i(y_{\phi(i)}, y_{\phi(-i)}, \overline{\gamma}, a, r_\alpha, r_\beta) = \int_0^1 E\left[u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, k_{\phi(i)}, r_{\widehat{\phi}(j)})\right] dj. \quad (2)$$

The arguments that we make explicit in this utility function, and in those appearing later, are those that are pertinent to the current stage of the game. Here, $\phi(-i)$ denotes the other platform, and the expectation is taken over the random parameters such as $\gamma_j$ and $k_{\phi(i)}$.

Platform $z \in \{\alpha, \beta\}$ charges consumers a connection fee of $p_z$. Consumers have a reservation utility of $R$ and consumers preferences are heterogenous, which we represent with the parameter $\theta_i$ that is uniformly distributed in the interval $[0, f]$. The total mass of consumers if $f$. Putting it all together, the utility of a consumer $i$ connecting to platform $\phi(i)$ is given by

$$U_i(\phi(i)) = \max\{R + \theta_i F_i(y_{\phi(i)}, y_{\phi(-i)}, \overline{\gamma}, a, r_\alpha, r_\beta) - p_{\phi(i)}, 0\}. \quad (3)$$

Consumers join the platform that yield the highest utility, provided it is positive.

**CP Profits**: If CPs connect to a platform $z \in \{\alpha, \beta\}$, they pay a fixed connection fee $w_z$ and make revenue by selling advertising and showing it to consumers. The utility $v_j$ of a CP $j$ is defined to be its profit:

$$v_j = V_j(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) - w_{\widehat{\phi}(j)}, \quad (4)$$

where the first term is its gross revenue, given by

$$V_j(\gamma_j, y_\alpha, y_\beta, q_\alpha, q_\beta) = \begin{cases} g(\gamma_j, y_\alpha)q_\alpha + g(\gamma_j, y_\beta)q_\beta & \text{if } \widehat{\phi}(j) = \alpha, \\ g(\gamma_j, y_\beta)q_\alpha + g(\gamma_j, y_\beta)q_\beta & \text{if } \widehat{\phi}(j) = \beta. \end{cases}$$

Here, $g(\gamma_j, y_{\widehat{\phi}(j)})$ is a function that represents ad prices. It is increasing in both parameters: Ad prices are high when content quality is good because it is easier to attract advertisers. In addition, consumers have a better experience with high-quality platforms and, therefore, they spend more time in these sites which increases the advertisers brand exposure. Thus advertisers are willing to pay more. Note that if CP $j$ joins the higher quality platform, it is able to charge a higher ad price for connections arising from consumers on that platform. If a CP joins the lower quality platform

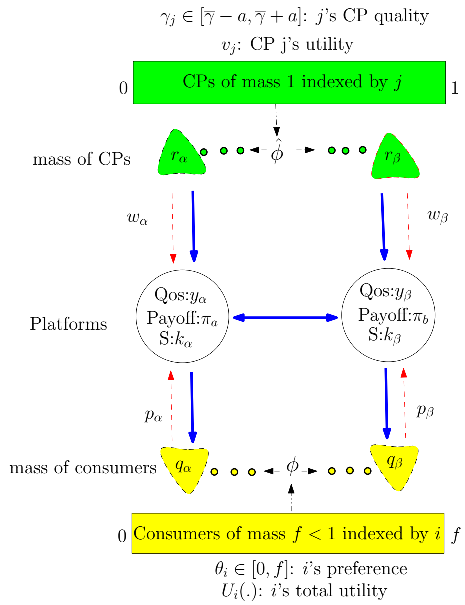


**Figure 2** **The dashed vertical lines show payments, while the solid thick lines refer to content going from CPs to consumers. The lines entitled $S$ within platforms refer to the additional services.**

its ad prices are the same across the two platforms because when a customer and CP connect to different platforms, the QoS is given by the worst of them.

**Platform Payoffs**: Finally we consider the platform payoff functions. Platforms pay for their quality investment, which is modeled with an increasing and convex investment cost $I(y_z)$ to achieve a QoS of $y_z$. This assumption results in decreasing returns to investment. Two additional, but standard, assumptions that we make for tractability are that the investment function is differentiable and that $I(0) = 0$. The payoff $\pi_z$ experienced by platform $z$ is given by

$$\pi_z = p_z q_z + w_z r_z - I(y_z). \tag{5}$$

To summarize all the players, elements and parameters, Figure 2 shows an illustration of the full model.

**Timing**: The dynamic game consists of the following stages:

1. Quality Investment Decisions: Platforms simultaneously choose QoS $y_\alpha$ and $y_\beta$.
2. CP Pricing Decisions: Platforms simultaneously choose fees $w_\alpha$ and $w_\beta$.
3. CP Connection Decisions: CPs decide which platform to join.
4. Consumer Pricing Decisions: Platforms simultaneously choose prices $p_\alpha$ and $p_\beta$.
5. Consumer Connection Decisions: Consumers decide which platform to join.
6. Consumer Consumption Decisions: Consumers decide which CPs to get service from.

The timing of the extensive game is predicated on the view that investments adjust more slowly than prices. The former is viewed as a medium to long-term decision whereas the latter is a shorter term decision. Thus investment is the first stage of the game. Prices for the CPs are set before those of the consumers to reflect the longer time horizon of the contracts between CPs and ISPs as opposed to those of consumers and ISPs. We solve this game by considering its subgame perfect Nash equilibrium (SPE), focusing on optimal actions/decisions along the equilibrium paths. To solve the game we use backward induction.

## 3. Model Analysis

Let $\mathcal{P} = \{\alpha, \beta, [0,1]_j, [0,f]_i\}$ denote the set of players in the multi-stage game, where $\alpha$ and $\beta$ are the platforms, and $[0,1]_j$ and $[0,f]_i$ are the continuum of CPs and consumers, respectively. We denote the information set at stage $k$ of the game for a player $\rho \in \mathcal{P}$ by $h^k_\rho$. Let the set of actions available to that player at that stage with that information set be denoted as $A_\rho(h^k_\rho)$. The main challenge to solve for an SPE in our model consists in solving the first three stages of the game. The analysis of the later stages of the game are more standard. Consumer prices at equilibrium follow from a standard vertical differentiation model (Tirole 1988). This analysis leaves us with a number of possible market configurations that could arise. To solve for the second stage, we first identify candidate Nash equilibrium CP price pairs for each of the market configurations. Then we show that these pairs are also best responses on the whole domain of strategies; i.e., a candidate price pair not only consists of prices that are mutual best responses in a particular market configuration but across all market configurations. To solve for the first stage of the game, we identify sets that contain the best responses and find their intersection points. These give us the candidate investment pairs. We then show that these pairs are indeed SPE by showing that neither of the platforms has an incentive to deviate. In the next subsections, we provide a more detailed analysis of each stage of the game.

### 3.1. Consumer Consumption Decisions

As usual with games of this kind, we begin the analysis with the last stage of the game where consumers select CPs with whom they will connect. A consumer $i$ on a platform $\phi(i) \in \{\alpha, \beta\}$ accessing content of a CP $j$ on platform $\widehat{\phi}(j) \in \{\alpha, \beta\}$ receives utility $u_{ij}$ represented in (1). As we discussed earlier, since $u_{ij} \geq 0$ for all consumer-CP pairs, when a consumer joins a platform he will connect to all CPs hosted by either platform.

### 3.2. Consumer Connection Decisions

In this stage of the game consumers decide which platform to join. The choice set of a consumer $i \in [0,f]$ given any $h^k_i$ is $A_i(h^k_i) = \{\alpha, \beta\}$. Through his information set, a consumer has knowledge of the number of CPs on each platform, the prices that platforms charge and the quality level of each platform. Each consumer $i$ maximizes his net utility given by (3) to determine what platform to connect to.

We assume that $y_\alpha > y_\beta$ and proceed to compute an allocation of consumers on each platform. Note that in this case $F_i(y_\alpha,\cdot) > F_i(y_\beta,\cdot)$. In Section 3.3, we will show that if $y_\alpha = y_\beta$, then any allocation of demand across platforms is possible at the resulting price equilibrium. We make the assumption that the reservation price $R$ is large enough so that the consumer market is covered. Indeed, because of (3) for large values of $R$ we have that $\theta_i > (p_{\phi(i)} - R)/F_i(y_{\phi(i)},.)$, implying that every consumer derives positive utility upon joining one of the platforms. We consider two disjoint cases to determine demand. If $p_\alpha < p_\beta$, consumers always join the platform with the highest perceived quality since $U_i(\phi(i)=\alpha) > U_i(\phi(i)=\beta)$, which follows directly from applying Lemma 1 in Appendix A.1. Hence, consumer demands are $q_\alpha = 1$ and $q_\beta = 0$.

The case of $p_\alpha \geq p_\beta$ is more involved. Let $\tilde{\theta} = (p_\alpha - p_\beta)/(F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot))$ be a threshold value. Consumers with a taste parameter $\theta_i \geq \tilde{\theta}$ join the platform with the higher perceived quality, $F_i(y_\alpha,\cdot)$, since $\theta_i F_i(y_\alpha,\cdot) - p_\alpha \geq \theta_i F_i(y_\beta,\cdot) - p_\beta$ if and only if $\theta_i \geq \tilde{\theta}$. Conversely, those whose taste parameter $\theta_i < \tilde{\theta}$ will join platform $\beta$. One can show that consumer demand is characterized by $q_\alpha(p_\alpha, p_\beta) = f - (p_\alpha - p_\beta)/(F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot))$ and $q_\beta(p_\alpha, p_\beta) = (p_\alpha - p_\beta)/(F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot))$.

### 3.3. Consumer Pricing Decisions

In this stage of the game platforms decide what prices $p_\alpha$ and $p_\beta$ to charge consumers. The choice set of platform $z \in \{\alpha, \beta\}$, given any $h_z^k$, is $A_z(h_z^k) = p_z \in \mathbb{R}_+$. Through its information set, a platform has knowledge of the number of CPs on each platform and the quality level of each platform. Profit for platform $z$ is given by (5). The equilibrium of this pricing subgame depends on the information set $h_z^k$. In particular, if $h_z^k$ is such that $y_\alpha > y_\beta$ it can be shown that $p_\alpha = 2f(F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot))/3$ and $p_\beta = f(F_i(y_\alpha,\cdot) - F_i(y_\beta,\cdot))/3$, and consumer demands at this equilibrium are $q_\alpha = 2f/3$ and $q_\beta = f/3$. If $h_z^k$ is such that $y_\alpha = y_\beta$ then $F_i(y_\alpha,\cdot) = F_i(y_\alpha,\cdot)$. Bertrand competition implies that the resulting subgame equilibrium is $p_\alpha = p_\beta = 0$. The consumer demands at this equilibrium price are arbitrary because any allocation such that $q_\alpha + q_\beta = f$ is a solution. In this case we make the standard assumption that consumers are evenly split between the platforms.

### 3.4. CP Connection Decisions

In this stage of the game, given the QoS $y_\alpha$ and $y_\beta$, and prices $w_\alpha$ and $w_\beta$ offered by platforms and anticipating the consumer mass on each of them, CPs decide on which platform to locate. The choice set of a CP $j$ given any $h_j^k$ is $A_j(h_j^k) = \{\text{none}, \alpha, \beta\}$. The utility $v_j$ extracted by a CP is given by (4) if it joins a platform or zero otherwise. Defining $g(\gamma_j, y_{\widehat{\phi}(j)}) = \gamma_j y_{\widehat{\phi}(j)}$, we have that the gross revenue earned by CP $j$ is $\gamma_j(y_z q_\alpha + y_\beta q_\beta)$ if they connect to platform $z$. CPs maximizes their utility $v_j$ and are indifferent between both platforms if and only if $\gamma_j(y_\alpha q_\alpha + y_\beta q_\beta) - w_\alpha = \gamma_j(y_\beta q_\alpha + y_\beta q_\beta) - w_\beta$. Letting $\widetilde{\gamma_j} = (w_\alpha - w_\beta)/(q_\alpha(y_\alpha - y_\beta))$ be the threshold, CPs with quality exceeding $\widetilde{\gamma_j}$ join the high-quality platform $\alpha$ and those with quality below it, but larger than $w_\beta/(y_\beta(q_\beta + q_\alpha))$, join the low-quality platform $\beta$. The rest do not join any platform. In our model, CPs participation in the market will be a proxy for CP innovation; if more CPs participate, more content is available for consumers.

Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, we define the following sets which correspond to the market configurations that may arise given a CP price pair $(w_\alpha, w_\beta)$. Here, the mass of CPs on each platform is written as a function of prices since the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ is known.

$$\begin{aligned}
\mathcal{R}_I =& \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) < 1,\, r_\alpha(w_\alpha, w_\beta) > 0,\, r_\beta(w_\alpha, w_\beta) = 0\},\\
\mathcal{R}_{II} =& \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) < 1,\, r_\alpha(w_\alpha, w_\beta) > 0,\, r_\beta(w_\alpha, w_\beta) > 0\},\\
\mathcal{R}_{III} =& \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) = 1,\, r_\alpha(w_\alpha, w_\beta) > 0,\, r_\beta(w_\alpha, w_\beta) > 0\},\\
\mathcal{R}_{IV} =& \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) + r_\beta(w_\alpha, w_\beta) = 1,\, r_\alpha(w_\alpha, w_\beta) = 1,\, r_\beta(w_\alpha, w_\beta) = 0\}.
\end{aligned}$$

We denote the market configurations corresponding to sets $\mathcal{R}_I$, $\mathcal{R}_{II}$, $\mathcal{R}_{III}$, and $\mathcal{R}_{IV}$ as $C_I$, $C_{II}$, $C_{III}$ and $C_{IV}$ respectively. The CP market is uncovered under the first two configurations, and covered under the last two.

### 3.5. CP Pricing Decisions

In this stage of the game platforms decide what prices to charge CPs. The choice set of platform $z \in \{\alpha, \beta\}$ given any $h_i^k$ is $A_i(h_i^k) = w_i \in \mathbb{R}_+$. Thus, platforms simultaneously decide what prices $w_\alpha$ and $w_\beta$ to charge to CPs.

We will show that in the SPE it is the case that $y_\alpha > y_\beta = 0$, that is, the low-quality platform does not invest. First, we characterize CP prices in the equilibrium path. In particular, we show that for any tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ for which $y_\alpha > y_\beta = 0$ there exists a unique SPE. Note that under this restriction on platforms qualities, CPs do not join the low-quality platform since they make no revenue, hence only configuration $C_I$ and $C_{IV}$ can be sustained.

Theorem 1. *Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ that satisfies that $y_\alpha > y_\beta = 0$, there exists a unique subgame perfect Nash equilibrium pair $(w_\alpha^*, w_\beta^*)$ in the price subgame. Moreover, the resulting market configuration is unique and the following statements hold:*

1. *If $1 < \frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f}$, then the equilibrium price pair $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{I}}$.*
2. *If $\frac{9+2f}{3+2f} \leq \frac{\overline{\gamma}}{a}$, then the equilibrium price pair $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{\mathcal{IV}}$.*

Theorem 1 allows us to conclude that we get a tipping equilibrium with all CPs locating in the platform with the highest quality when the low-quality platform does not invest. We prove the existence of the price SPE constructively. To do that, we first identify candidate equilibrium price pairs in each possible market configuration (see Appendix A.2), and then check whether these price equilibrium pairs are indeed Nash equilibria of the price subgame (see Appendix A.3). We do so by verifying that the equilibrium price candidates are best replies on the whole domain of strategies; that is, not only they are best responses in their respective market configurations but also best replies if the other market configurations are taken into account.

In Appendix A.3.1, we provide a complete characterization of the CP price SPE given any tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ for which $y_\alpha > y_\beta > 0$. In this case, we show that the uncovered market configuration $C_I$ does not occur at an SPE. On the other hand, we show that given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ one of the other configurations, $C_{II}$, $C_{III}$ or $C_{IV}$, will emerge. In doing so, we determine the set of parametric values $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ for which these different configurations exist. We believe that the characterization of equilibrium in the case $y_\alpha > y_\beta \geq 0$ is of interest by itself, but we have omitted details here and include them in the appendix for brevity. Finally, Lemma 10 in Appendix A.5 shows that when $y_\alpha = y_\beta$ an SPE does not exist.

### 3.6. Quality Investment Decisions

In this stage of the game platforms simultaneously decide how much to invest in quality. The choice set of platform $z \in \{\alpha, \beta\}$ given any $h_z^k$ is $A_z(h_z^k) = y_z$ where $y_z \in \mathbb{R}_+$. We show that a unique SPE exists. In addition, we show that this equilibrium involves maximal differentiation subject to investment costs: one platform invests in the highest quality possible taking into account investment costs while the other chooses not to invest. Moreover, we characterize the investment levels in terms of the market parameters. We find the equilibrium quality choices by considering sets that contain the best responses for both platforms. We show that the equilibria are given by the intersection of these sets. The following theorem shows the necessary conditions for the existence of an SPE. It enables us to identify candidate equilibrium investment pairs when the mass of consumers is above a critical level.

Theorem 2. *Assume that $f \geq 3/5$ and $I'(0)$ is small enough. If an SPE exists in the quality investment game, then one platform does not invest in quality and the other makes a positive investment of $y^*(\overline{\gamma}, a, f)$. Moreover, the equilibrium investment level is characterized by*

$$I'(y^*) = \begin{cases} I_1(\overline{\gamma}, a, f) & \text{if } \frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f} \\ I_2(\overline{\gamma}, a, f) & \text{if } \frac{\overline{\gamma}}{a} \geq \frac{9+2f}{3+2f} \end{cases}$$

*where $I_1(\overline{\gamma}, a, f) = (4(\overline{\gamma} - a)^2 f^3 + 12(\overline{\gamma} + a)(\overline{\gamma} + 3a) f^2 + 9(\overline{\gamma} + a)^2 f)/108a$, and $I_2(\overline{\gamma}, a, f) = 2f(\overline{\gamma}(3 + 4f) - 3a)/9$.*

The next theorem shows that the characterization above is indeed an SPE when investment functions are quadratic. Because we prove existence of equilibrium constructively, assuming quadratic costs simplifies the analysis.

Theorem 3. *Let the investment cost function be of the form $I(y) = cy^2$ (which satisfies the assumptions on $I(\cdot)$) and $f \geq \max\{3/5, 1 - a/\overline{\gamma}\}$. Then, the quality investment game has an SPE.*

The results above suggest that platforms differentiate in quality to soften price competition. If platforms are undifferentiated, they earn zero profits due to the ensuing Bertrand price competition on both sides of the market. Therefore, at equilibrium, platforms have the incentive to invest in different quality levels and achieve maximum differentiation. Recall that the lack of investment is interpreted as the minimal investment needed to have an operational platform.

## 4. Investment and Market Coverage in the Neutral Case

Having analyzed all the stages of the game we now discuss the investment levels and CP market coverage at the SPE in the neutral model. On both sides of the market the platforms are viewed as substitute products by both consumers and CPs. Thus platforms make higher profits when they are more differentiated. The high-quality platform gains by investing more and the low-quality platform by investing less. For the low-quality platform the differentiation not only gives it market power on the consumer side but also reduces its investment cost. Indeed, investment by the low-quality platform increases competition on the consumer side in addition to increasing investment cost, resulting in lower consumer prices and consequently platform's profit. This reduction is larger than the additional revenues extracted from CPs this investment would generate.

The investment level of the high-quality platform increases with CPs' average quality. This increases the revenues that CPs earn; recall that the advert price is increasing in platform quality. Thus the surplus from which the high-quality platform can extract revenue also increases. Note that for a given investment level, the surplus from which the high-quality platform can extract revenue is larger when CPs' qualities increase, enhancing investment incentives. In contrast, as shown in Figure 3, the relationship between the investment level and the heterogeneity is unimodal and convex. An increase in heterogeneity generally makes demand of CPs less elastic. Hence, the high-quality platform prefers to make revenue directly by raising prices rather than through investment which is more costly. However, as heterogeneity increases beyond a critical point the platform prefers to invest in quality. Due to the high prices, the CP market becomes progressively uncovered. To gain revenue from the diminishing CP base, the high-quality platform invests to increase the surplus from which it can expropriate revenue.

We next present a corollary of Theorem 2 that characterizes market coverage by CPs at the SPE.

Corollary 1. *Assume that $f \geq 3/5$. In the SPE, all CPs connect to the high-quality platform, and the market is covered if and only if $\overline{\gamma}/a \geq (9+2f)/(3+2f)$.*

When $\overline{\gamma}/a$ is low, the outcome of the game is that all CPs flock the high-quality platform without covering the entire market, that is, there is a mass of CPs that are not active in the equilibrium.

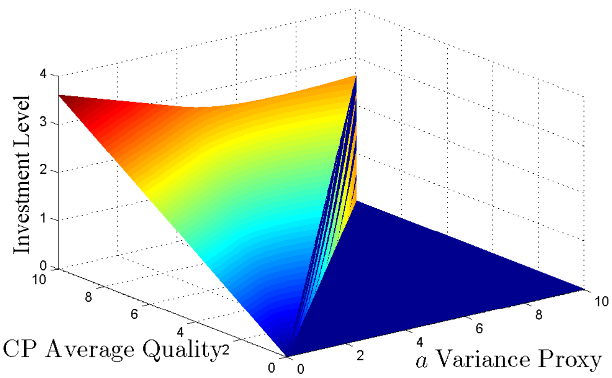


**Figure 3** **Investment level of the high-quality platform as a function of $\overline{\gamma}$ and $a$.**

In that case, either $a$ is high which implies that CP demand is less elastic which leads to higher prices for the CPs and less enrollment, or $\overline{\gamma}$ is low which implies that low quality CPs do not earn enough revenues to join the platform. Instead, for high values of $\overline{\gamma}/a$, the market is covered, that is all CPs are active and participate in the equilibrium. In this case, either $\overline{\gamma}$ is high or $a$ is low. In the former case CPs earn high advertising revenues and thus all CPs can afford to join the platform. In the case of a low variance, CPs demand is more elastic. Therefore prices charged to CPs are low encouraging high enrollment.

## 5. The Non-Neutral Model and its Analysis

To study the non-neutral regime, we employ a model that is equal to that in the neutral regime except for one important difference: if a CP wants to reach the customers in one platform, it must pay that platform for that access, and if the CP wants to reach all customers, then it must pay both platforms. As in the neutral case, each platform $z$ charges a fixed connection fee $w_z$. All other aspects are the same. Platforms invest in quality, CPs earn revenue by selling advertising, and consumers connect to one of the two platforms. We solve for the SPE of this game, which we find using backward induction, and compare it to the solution of the neutral model. Without loss of generality, we continue with the assumption that $y_\alpha \geq y_\beta \geq 0$. In the next subsections, we provide a more detailed analysis of each stage of the game, now for the non-neutral case.

### 5.1. Consumer Consumption Decisions

In the last stage of the game, consumers select CPs with whom they will connect. A consumer $i$ on a platform $\phi(i)$ connects to a CP $j$ only if the CP bought access to $\phi(i)$. Thus the utility gained by the consumer connecting to the CP is given by $u_{ij}(y_{\phi(i)}, \gamma_j, k_{\phi(i)}, r_{\phi(i)}) = y_{\phi(i)}(\gamma_j/r_{\phi(i)} + k_{\phi(i)})$. Since this value is non-negative, all consumers will select all CPs that are accessible.

### 5.2. Consumer Connection Decisions

In this stage of the game consumers choose a platform to join. The quality perceived by a consumer $i$ when he joins platform $\phi(i)$ is given by $F_i(y_{\phi(i)}, \overline{\gamma}, a, r_{\phi(i)}) = \int_0^1 E\left[u_{ij}(y_{\phi(i)}, \gamma_j, k_{\phi(i)}, r_{\phi(i)})\right] dj$. The utility is given by $U_i(\phi(i)) = R + \theta_i F_i(y_{\phi(i)}, \overline{\gamma}, a, r_{\phi(i)}) - p_{\phi(i)}$, where again we have assumed that $R$ is large enough so that the consumer market is covered. Given an information set $h_i^k$ one of the following three relations hold: (i) $F_i(y_\alpha, r_\alpha, a, \overline{\gamma}) > F_i(y_\beta, r_\beta, a, \overline{\gamma})$, (ii) $F_i(y_\alpha, r_\alpha, a, \overline{\gamma}) < F_i(y_\beta, r_\beta, a, \overline{\gamma})$, (iii) $F_i(y_\alpha, r_\alpha, a, \overline{\gamma}) = F_i(y_\beta, r_\beta, a, \overline{\gamma})$. Platforms demands $q_\alpha$ and $q_\beta$ are derived as in Section 3.2, based on the prices offered by platforms and on which of the above relations holds. Note that in the non-neutral model, even though $y_\alpha \geq y_\beta$, relation (ii) may hold for some values of $r_\alpha$ and $r_\beta$; this never happens in the neutral model. This introduces additional complexity in the analysis of the non-neutral model as we discuss below.

### 5.3. Consumer Pricing Decisions

In this stage of the game platforms simultaneously decide what prices to charge to the consumers. Information sets in this stage can be classified into three types depending the three relations of Section 5.2. We characterize prices at equilibrium for each relation.

When (i) holds, the resulting consumer prices are $p_\alpha = q_\alpha(F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot))$ and $p_\beta = q_\beta(F_i(y_\alpha, \cdot) - F_i(y_\beta, \cdot))$ and consumer demands are $q_\alpha = 2f/3$ and $q_\beta = f/3$. When (ii) holds, a symmetric characterization applies. Last, when (iii) holds then $p_\alpha = p_\beta = 0$. We make the standard assumption that consumers are split evenly. The analysis is similar to that in Section 3.3.

### 5.4. CP Connection Decisions

In this stage of the game CPs simultaneously decide which platforms to join. A CP $j$ has a choice set $A_j(h_j^k) = \{\text{none}, \alpha, \beta, \text{both}\}$, and makes the decision given the pair of QoS $(y_\alpha, y_\beta)$ and the pair of prices $(w_\alpha, w_\beta)$. We can view a CP as having an option to buy one of three possible types of connection services. Defining $g(\gamma_j, y_{\widehat{\phi}(j)}) = \gamma_j y_{\widehat{\phi}(j)}$ as before, CP profits are

$$v_j = \begin{cases} g(\gamma_j, y_\alpha) q_\alpha - w_\alpha & \text{if } \widehat{\phi}(j) = \alpha, \\ g(\gamma_j, y_\beta) q_\beta - w_\beta & \text{if } \widehat{\phi}(j) = \beta, \\ g(\gamma_j, y_\alpha) q_\alpha + g(\gamma_j, y_\beta) q_\beta - w_\alpha - w_\beta & \text{if } \widehat{\phi}(j) = \text{both}. \end{cases}$$

A CP $j$ is willing to join both platforms if $\gamma_j \geq (w_\alpha + w_\beta)/(y_\alpha q_\alpha + y_\beta q_\beta)$. For an exclusive connection to platform $z$, a CP $j$ is willing to join it if $\gamma_j \geq w_z/(y_z q_z)$. Given a price pair $(w_\alpha, w_\beta)$, together with the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, we refer to the resulting CP demand on each platform as the CP allocation equilibrium. A CP allocation equilibrium also determines which of the relations in Section 5.2 hold on the equilibrium path. In Appendix B, we derive the sets of prices $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$ and $\mathcal{W}_{R(iii)}$ for which the CP allocation equilibrium leads to relations $(i)$, $(ii)$ and $(iii)$ holding on the equilibrium path. Note that, if a price pair lies on the intersection of any of the sets $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$, and $\mathcal{W}_{R(iii)}$, then more than one CP allocation equilibrium exists. The CP demand faced by platform $z \in \{\alpha, \beta\}$ is given by $r_z = \max\{\min\{1, (\overline{\gamma} + a - w_z/(q_z y_z))/(2a)\}, 0\}$, where $q_z$ depends on which of the relations holds on the equilibrium path.

### 5.5. CP Pricing Decisions

In this stage of the game platforms decide what prices to charge to CPs. The multiplicity of CP allocation equilibria mentioned above makes the analysis of this stage challenging. For tractability, in the remaining sections, we focus only on price games that result when the CP allocation equilibria selected (if multiple equilibria exist in the price subgames) are such that either relation (i) or (iii) hold. We formalize this in the following assumption.

Assumption 1. *Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta, w_\alpha, w_\beta)$ for which $y_\alpha \geq y_\beta \geq 0$ such that multiple CP allocation equilibria exist in the price subgame, we assume that only equilibria that yield relations (i) or (iii) are selected.*

This assumption intuitively implies that CPs will anticipate that more consumers will join the platform with the larger investment in quality (recall that we have assumed $y_\alpha \geq y_\beta$). In addition, our assumption is partially motivated by the fact that if an SPE exists in one of the CP price games then the CP allocation equilibrium on the equilibrium path does not yield relation (ii), see Appendix B.2. Note, however, that the assumption is still needed to analyze CP price games that are off-the-equilibrium path.

The next theorem characterizes an equilibrium for CP prices in the case of $y_\alpha > y_\beta$. The proof, price characterizations and conditions for various market configurations to exist are given in Appendix B.3. There we also show that the market configuration depends only on the heterogeneity parameter $a$, the average CP quality $\overline{\gamma}$ and the consumer mass $f$.

Theorem 4. *Let Assumption 1 hold. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta$, the price-subgame admits a unique SPE pair $(w^*_\alpha, w^*_\beta)$. Moreover, the resulting market configuration is unique.*

We show in Appendix B.4 that if an SPE in prices exists when $y_\alpha = y_\beta$, the platforms have an incentive to deviate; therefore, symmetric investment levels cannot be sustained in an SPE.

### 5.6. Quality Investment Decisions

In this stage of the game platforms simultaneously decide how much to invest in quality. We assume that investment costs are quadratic, equal to $cy^2$ with $c \geq 1$. We find the equilibrium quality choices by considering the best reply responses of the two platforms. We find the set that contains platform $\beta$'s best replies to platform $\alpha$'s choices and viceversa, and establish that the best reply functions intersect at a unique point. This proves that there is a unique SPE in the investment game. As a corollary, in Section 6 we characterize the resulting market configurations.

To simplify the presentation, we let $t_1 = (9+2f)/(3+2f)$, $t_2 = (f^2+12f-9+4\sqrt{3f^3})/(-6f+9+f^2)$ and $t_3 = (9-f)/(3-f)$ and define the following regions:

$$\mathcal{R}_1 = \{1 < \overline{\gamma}/a \leq \min\{t_1, t_2\}\}, \qquad \mathcal{R}_2 = \{\max\{1, t_2\} < \overline{\gamma}/a < t_1\},$$

$$\mathcal{R}_3 = \{t_1 < \overline{\gamma}/a < t_2\}, \qquad \mathcal{R}_4 = \{\max\{t_1, t_2\} \leq \overline{\gamma}/a \leq t_3\},$$

$$\mathcal{R}_5 = \{t_3 < \overline{\gamma}/a \leq \infty\}.$$

These regions are broadly classified according to the market configurations (as defined in Appendix B.3) that arise at the SPE given the tuple $(\overline{\gamma}, a, f)$. Qualitatively, the partitions represent regions in which the heterogeneity in content quality is either high ($\mathcal{R}_1$, $\mathcal{R}_2$), medium ($\mathcal{R}_3$, $\mathcal{R}_4$) or low ($\mathcal{R}_5$). The following theorem characterizes the investment levels at equilibrium; its proof can be found in Appendix B.5. The functions $I_1(\overline{\gamma}, a, f)$, and $I_2(\overline{\gamma}, a, f)$ are defined as in Theorem 2.

Theorem 5. *Let Assumption 1 hold. Given a tuple $(\overline{\gamma}, a, c)$ and $f > 0.47$ there exists a unique subgame perfect Nash equilibrium (SPE) in the quality investment game. Moreover, the following statements hold:*

1. *If $\mathcal{R}_1$ holds then the SPE entails one platform investing in positive quality $y^*(\overline{\gamma}, a, f, c) = I_1(\overline{\gamma}, a, f)/2c$, and the other not investing in any quality.*
2. *If $\mathcal{R}_2$ holds then the SPE entails both platforms investing in positive qualities where one invests in a higher quality $y_h^*(\overline{\gamma}, a, f, c) = I_1(\overline{\gamma}, a, f)/2c$, and the other invests in a lower quality $y_l^*(\overline{\gamma}, a, f, c) = ((\overline{\gamma} - a)^2 f^3 - 6(\overline{\gamma} + a)(\overline{\gamma} + 3a)f^2 + 9(\overline{\gamma} + a)^2 f)/(432ca)$.*
3. *If $\mathcal{R}_3$ holds then the SPE entails one platform investing in positive quality $y^*(\overline{\gamma}, a, fc) = I_2(\overline{\gamma}, a, f)/2c$, and the other not investing in any quality.*
4. *If $\mathcal{R}_4$ holds then the SPE entails both platforms investing in positive qualities: one invests in a higher quality $y_h^*(\overline{\gamma}, a, f, c) = I_2(\overline{\gamma}, a, f)/2c$, and the other invests in a lower quality $y_l^*(\overline{\gamma}, a, f, c) = ((\overline{\gamma} - a)^2 f^3 - 6(\overline{\gamma} + a)(\overline{\gamma} + 3a)f^2 + 9(\overline{\gamma} + a)^2 f)/(432ca)$.*
5. *If $\mathcal{R}_5$ holds then the SPE entails both platforms investing in positive qualities: one invests in a higher quality $y_h^*(\overline{\gamma}, a, f, c) = I_2(\overline{\gamma}, a, f)/2c$, and the other invests in lower quality $y_l^*(\overline{\gamma}, a, f, c) = f(3\overline{\gamma} - 2f\overline{\gamma} - 3a)/18c$.*

We impose the condition $f > 0.47$ since for smaller values of $f$ an SPE in the quality investment game may not always exist in all the regions. The implications of the theorem are discussed in the next section.

## 6. Investment and Market Coverage in the Non-Neutral Case

In the non-neutral regime, platforms are substitutes only on the consumer side of the market. On the CP side, CPs make decisions regarding whether to join each platform independently from the other platform. In this case, a platform has monopolistic power over CPs since its only through it that CPs can connect to its subscribed consumers. Thus investment decisions observed in this regime are driven by the trade-off platforms make between differentiating in quality to make revenue on the consumer side and exerting their monopoly power over the access to their consumer bases to extract revenue from CPs.

Given the tuple $(\overline{\gamma}, a, f)$, the level of investment of the high-quality platform in the SPE is the same as that in the neutral regime and varies with the average and heterogeneity of CP quality in a similar way. Quality investment in the low-quality platform also increases with the average CP quality for the same reasons as those highlighted for the high-quality platform.

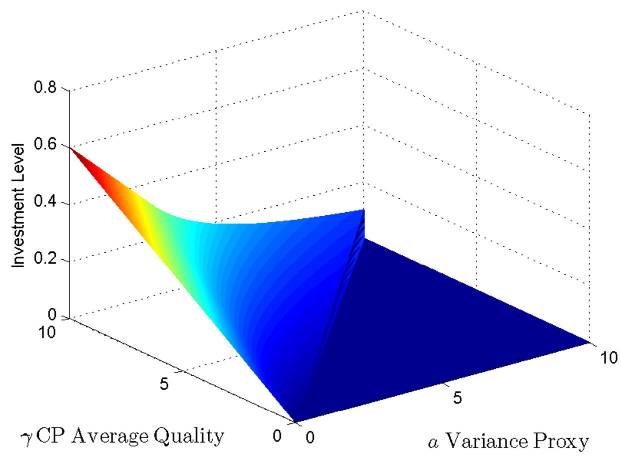


**Figure 4** **Investment level of the low-quality platform as a function of $\overline{\gamma}$ and $a$.**

As the heterogeneity of CPs quality increases, the value of investment of the low-quality platform decreases, this is in contrast to the behavior exhibited by the high-quality platform, see Figure 4. An increase in heterogeneity leads to a less elastic CP demand, hence platforms prefer to extract CP revenue through a price increase than investing in quality which is costly. Even though the low-quality platform can gain from increased CP surplus with a larger quality investment, (since increased platform quality increases advert revenues) the increase in competition on the consumer side would offset this gain.

We now explore the investment patterns as a function of the consumer mass and the heterogeneity in content quality, see Figure 5. When $\overline{\gamma}/a$ has a low value and $f$ is high, i.e. in the regions denoted by $\mathcal{R}_1$ and $\mathcal{R}_3$ the platforms differentiate as much as possible. One platform invests in a positive quality while the other opts not to invest. Similar to the neutral case, both platforms make more profit when they are more differentiated. A low value of $\overline{\gamma}/a$ is primarily driven by low average CP quality. Therefore the advertising revenue gained by CPs is also low. Consequently, if the low-quality platform invests, the profits expropriated from CPs would not be enough to offset the costs of investment plus the loss of revenue caused by the heightened intensity in competition in the consumer side. Moreover, since $f$ is large, the mass of consumers joining the platforms is higher

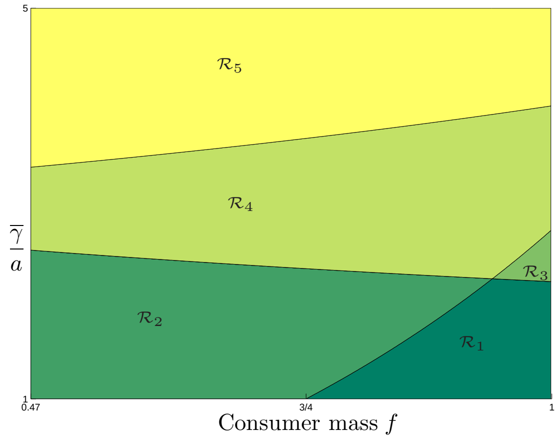


**Figure 5 Market Coverage and Connection Decisions as a function of $\overline{\gamma}/a$ and $f$.**

which further increases the revenue made from the consumer side and dissuades the platform from investing.

In contrast when $f$ and $\overline{\gamma}/a$ are both low, the two platforms invest in quality, although not at the same level. In this region, denoted by $\mathcal{R}_2$, since $f$ is small, by investing the platform gains more from the CP side than the revenue loses caused by competition on the consumer side. Where $\overline{\gamma}/a$ has a medium to high value, regions $\mathcal{R}_4$ and $\mathcal{R}_5$, the platforms partially differentiate. As in the previous case, both platforms invest at different levels.

We have the following important corollary. Below, high quality CPs are such that their quality is above a threshold that depends on the model parameters.

Corollary 2. *In the SPE the following holds. If $\mathcal{R}_1$ or $\mathcal{R}_3$ holds, CPs are hosted exclusively by the high-quality platform. If $\mathcal{R}_2$ or $\mathcal{R}_4$ holds, then only the high-quality CPs connect to both platforms and the rest of CPs connect exclusively to the high-quality platform. Otherwise (for $\mathcal{R}_5$) all CPs connect to both platforms.*

*Moreover, if $\mathcal{R}_1$ or $\mathcal{R}_2$ holds then the outcome is given by an uncovered CP market. Otherwise (for $\mathcal{R}_3$, $\mathcal{R}_4$ or $\mathcal{R}_5$) the CP market is covered, that is all CPs connect to at least one platform.*

In region $\mathcal{R}_1$, CPs exclusively join the high quality platform and the market is uncovered. Observe that in this region the low-quality platform does not invest. Therefore there is no value to be gained by a CP joining the lower quality platform. Region $\mathcal{R}_2$ also represents an uncovered market but CPs patronize both platforms with low-quality CPs being exclusive to the high-quality platform and high-quality CPs joining both. Observe that only the high-quality CPs connect to both platforms since they earn enough advertising revenue to cover the cost of connecting to both platforms. In the remaining regions the market is covered since all the CPs in the market serve their content through the high-quality platform. This is because for medium to high values of $\overline{\gamma}/a$ CPs earn higher advertising rates on the high-quality platform. Lack of investment, in region $\mathcal{R}_3$, by the low-quality platform leads to CPs not joining it since they will not make any advertising revenue. In contrast, low-quality platforms have CPs subscribe to them in region $\mathcal{R}_4$. In this case, investment by the low-quality platform is attractive to the high-quality CPs who can command higher advertising prices that offset costs of joining the two platforms. Finally, in region $\mathcal{R}_5$ all CPs patronize both platforms. The average CP quality is high enough and the variation of CP quality low enough that the advertising prices the CPs command enable them to gain more value when they connect to both than when they connect to only one platform.

## 7. Comparison of the Two Regimes

In this section we compare the neutral and non-neutral regimes with respect to social welfare, and surplus and profits extracted by consumers and CPs. We first define social welfare and its constituent parts. We show that CP and consumer surplus in the non-neutral model are at least equal to, if not superior than, that in the neutral model. In addition, we show a dichotomy of preferences for the two regimes by platforms. The low-quality platform prefers the non-neutral regime while the high-quality one prefers the neutral regime. Finally, we show that given a tuple $(\overline{\gamma}, a, f)$ the non-neutral regime results in at least as high social welfare than that in the neutral regime, and strictly higher for a significant range of parameters.

**Participants' Utilities.** Social welfare is defined as the sum of all the participants' utilities:

1. Consumer surplus: A consumer $i \in [0, f]$ subscribing to platform $\phi(i)$ has an expected utility given by $E[U_i] = E[R + \theta_i F_i(y_{\phi(i)}, \cdot) - p_{\phi(i)}]$ (see (3)). The aggregate consumer surplus equals $\int_0^f E[U_i]di = \sum_{z\in\{\alpha,\beta\}} q_z(R + E[\theta_i|\phi(i) = z]F_i(y_z, \cdot) - p_z)$.

2. Platforms' profit: The profit for a platform $\pi_z$ is displayed in (5). The total profit among platforms equals $\sum_{z\in\{\alpha,\beta\}} p_z q_z + r_z w_z - c y_z^2$.

3. CPs' Profit: A CP $j \in [0, 1]_j$ hosted by platform $\hat{\phi}(j)$ has expected profits given by $E[v_j]$ (see (4)). The aggregate CP surplus equals $\int_0^1 \sum_{z\in\{\alpha,\beta\}} E[v_j \mid \hat{\phi}(j) = z] r_z dj$ in the neutral regime, and $\int_0^1 \sum_{z\in\{\alpha,\beta,\text{both}\}} E[v_j \mid \hat{\phi}(j) = z] r_z dj$ in the non-neutral case.

First, we revise and compare investments and market coverage in both models.

**Investments.** Recall that in the SPE of the neutral model the high-quality platform invests a positive amount and the low-quality platform does not invest. In the SPE of the non-neutral model, the investment level of the high-quality platform is the same as in the neutral model for all parameter values. The main difference between both models, that will drive the comparisons below, is that the low-quality platform invests a positive amount for a significant range of parameter values.

**Market Coverage.** Our proxy for CP innovation is CP market coverage. For all market values, CP market coverage in the SPE is the same for both models. In particular, the mass of CPs that connects through the high-quality platform is the same. The main difference between both models, however, is that in the non-neutral model and for some parameter values a mass of CPs will also connect through the low-quality platform. Note that in contrast, because the low-quality platform does not invest in the neutral model, CPs do not connect to it.

**CP and Consumer Surplus Comparison.** CP surplus is at least as high in the non-neutral model compared to that in the neutral model, see Appendix C.2. Moreover, it is strictly higher in the non-neutral regime for all parameter values for which the low-quality platform makes positive investments. In particular, investments by the low-quality platform increase the advertising revenue earned by CPs that connect to it. Consumer surplus is also higher in the non-neutral regime in the cases where the low-quality platform invests in quality, see Appendix C.3. This investment increases consumer surplus for two reasons. First, an increase in the low-quality platform investment level intensifies price competition on the consumer side resulting in lower prices. Therefore, consumers are able to keep more of their surplus. Second, an increase in platform quality increases the value gained by consumers who join the low-quality platform because this enhances CP quality.

**Platform Profits Comparison.** If the low-quality platform makes positive investments, profits for platforms differ in both regimes. In particular, profits of the high-quality platform are higher in the neutral regime. In this regime the platforms are maximally differentiated, and the high-quality platform serves high-quality consumers and CPs and extracts more revenue from both due to the resulting market power that arises from differentiation. In the non-neutral regime, the investment by the low-quality platform results in more intense price competition on the consumer side that reduces the high-quality platform's overall profits. In contrast, the low-quality platform's profits are superior in the non-neutral regime. Note that in the neutral regime, the low-quality platform makes revenue only on the consumer side, while in the non-neutral regime, it makes revenue from both sides of the market. Although the investment by the low-quality platform intensifies competition on the consumer side and reduces revenue, it enables the platform to increase the revenue from CPs which offsets losses due to this competition. Overall, aggregate profit in the neutral regime is higher than that in the non-neutral regime, because price competition is softer (see Appendix C.4).

**Social Welfare Comparison.** Adding up the previous effects, we conclude that in SPE, social welfare in the non-neutral model is at least as high compared to that in the neutral model, see Appendix C.1. Moreover, social welfare in the non-neutral model is strictly higher when the low-quality platform invests. A key driver of this result is the increase of consumer surplus and CP profits in the non-neutral regime. As an example, Figure 6 shows the welfare difference between the non-neutral and the neutral regimes for $f = 0.6$ and $c = 1$. Since the welfare difference is driven by the investment level of the low-quality platform in the non-neutral regime, for a fixed heterogeneity $a$, the difference is increasing when $\overline{\gamma}$ increases (see Section 6). On the other hand, an increase in $a$ results in a lower welfare difference. This again reflects the effects of heterogeneity on investments in the low-quality platform.

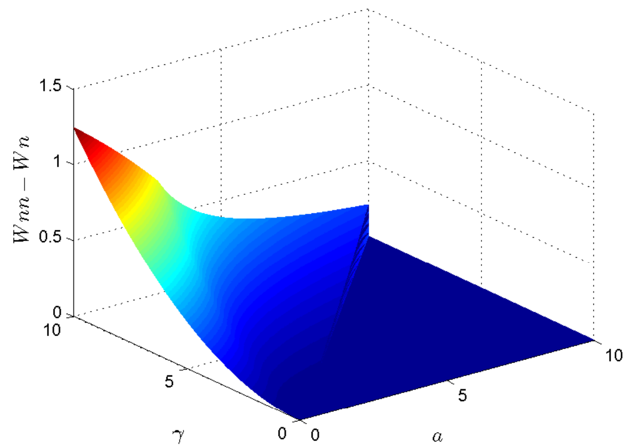


**Figure 6 Welfare difference between the non-neutral and neutral regimes.**

## 8. Conclusion

We have analyzed a model of the Internet that explicitly incorporates the competition among ISPs, CPs and consumers. We have looked at two versions of the model that employ different contracts between CPs and ISPs to contribute to the net neutrality debate through formal economic analysis. We have explored the effect that the different contracts have on social welfare, platform profits, and consumer and CP surplus. One important finding is that under our assumptions the non-neutral model leads to higher aggregate levels of investment because the low-quality platform invests in the non-neutral model but not in the neutral one. Contrary to qualitative arguments that are found in the literature (Wu and Yoo 2007, Lee and Wu 2009, see, e.g.,), our results suggest that access fees—payments by off-net CPs to ISPs in order to access consumers—could positively impact investment incentives leading to upgrades of existing network infrastructure. Moreover, in contrast to some results in the literature such as Economides and Tåg (2007), Cañon (2009), we find that social welfare is generally superior in the non-neutral regime. This follows because the aggregate level of investment is higher, increasing both CP and consumer surplus.

These results together suggest that price regulation could possibly be an inapt policy to increase value in the Internet, because it could limit the investment incentives of smaller ISPs or network providers. Low investments by these ISPs would decrease CPs and consumer utility directly through low QoS and indirectly through bottleneck effects. Therefore, if the goal is creating value and having both consumers and CPs enjoy a high welfare, it is important to establish policies that foster investments.

While our results are suggestive, our model is obviously stylized so they need to be taken with caution. One simplifying feature of our analysis is the lack of transaction costs in the non-neutral regime. These present new analytical challenges but are an important area to explore because they would reduce the revenue earned by the platforms and thus likely temper (perhaps in a negative way) the investment incentives of ISPs. Another direction of future research is the explicit modeling of quality investment by CPs. An interesting modification would have CP quality being determined endogenously and investigate CP incentives under both models. This extension, which would also involve significant technical challenges, could possibly change some of our qualitative conclusions. We leave these ideas and extensions for future research.

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# Appendix A: Technical Details for the Neutral Model

## A.1. A Lemma

The following lemma shows that if a platform has a higher platform quality in the neutral regime then consumers joining it perceive it to be of a higher value.

LEMMA 1. *If $y_\alpha > y_\beta$, then $F_i(y_\alpha,\cdot) > F_i(y_\alpha,\cdot)$.*

*Proof.* After some algebraic manipulations the explicit expression for $F_i(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \overline{\gamma}, r_\alpha, r_\beta)$ in terms of average CP content quality, platform quality and the mass of content providers on both platforms is as given below for both $\phi(i)=\alpha$ and $\phi(i)=\beta$;

$$
\begin{aligned}
F_i(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \overline{\gamma}, r_\alpha, r_\beta) =& \int_0^1 E\left[\max\{u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, c_{\phi(i)}, r_{\widehat{\phi}(j)}),0\}\right] dj,\\
=& y_\alpha(\overline{\gamma}(r_\alpha+1)+a(1-r_\alpha)) + y_\beta(\overline{\gamma}(r_\beta+1)+a(1-r_\alpha-r_\beta)),\\
F_i(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \overline{\gamma}, r_\alpha, r_\beta) =& \int_0^1 E\left[\max\{u_{ij}(y_{\phi(i)}, y_{\widehat{\phi}(j)}, \gamma_j, c_{\phi(i)}, r_{\widehat{\phi}(j)}),0\}\right] dj,\\
=& y_\beta(\overline{\gamma}(r_\alpha+1)+a(1-r_\alpha)) + y_\beta(\overline{\gamma}(r_\beta+1)+a(1-r_\alpha-r_\beta)).
\end{aligned}
$$

It immediately follows from above that $F_i(y_\alpha,\cdot) > F_i(y_\alpha,\cdot)$. □

## A.2. Candidate Equilibrium Prices for Different Markets.

**Uncovered Market - $C_I$.** In this case we suppose *ex ante* that the market is uncovered with only the high quality platform serving the market. We identify the equilibrium prices for this market configuration and the conditions on $(\overline{\gamma}, a, f)$ for which this market configuration is feasible. We first derive the best price responses of each platform to the price set by its rival. The condition for an uncovered market where only the high-quality platform participates in the market is given by,

$$\frac{w_\beta}{(q_\beta+q_\alpha)y_\beta} \geq \frac{w_\alpha}{y_\beta q_\beta + y_\alpha q_\alpha} > \overline{\gamma} - a. \tag{6}$$

In this configuration, both platform's profit do not depend on $w_\beta$. Thus given $w_\alpha$ that satisfies $(w_\alpha)/(y_\beta q_\beta + y_\alpha q_\alpha) > \overline{\gamma} - a$, any $w_\beta$ that satisfies the following condition

$$w_\beta \geq y_\beta \frac{w_\alpha(q_\alpha+q_\beta)}{y_\beta q_\beta + y_\alpha q_\alpha}$$

is a best response by platform $\beta$. This follows from condition (6). On the other hand given $w_\beta > (\overline{\gamma}-a)(q_\alpha + q_\beta)y_\beta$ the best response is given by the optimal solution of the following problem

$$
\begin{aligned}
&\max \pi_\alpha^{ui}(w_\alpha, w_\beta) \\
&\text{s.t. } w_\alpha \in \left( (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha), \frac{w_\beta(y_\beta q_\beta + y_\alpha q_\alpha)}{y_\beta(q_\alpha+q_\beta)} \right].
\end{aligned}
\tag{7}
$$

From the first order conditions of (7) we infer that the best response is characterized as follows,

$$w_\alpha = \begin{cases} w_\alpha^* & \text{if } w_\beta \geq \frac{w_\alpha^* y_\beta(q_\alpha+q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)} \\ \frac{w_\beta(y_\beta q_\beta + y_\alpha q_\alpha)}{y_\beta(q_\alpha+q_\beta)} & \text{if } w_\beta < \frac{w_\alpha^* y_\beta(q_\alpha+q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)}. \end{cases}$$

where $w_\alpha^* = \frac{f}{9}(5a+\overline{\gamma})y_\alpha - \frac{f}{18}(a-7\overline{\gamma})y_\beta$ and is the unrestricted solution to problem 7. Denoting the equilibrium price pair in this configuration by $(w_\alpha^{ui}, w_\beta^{ui})$ we note that any price combination,

$$\frac{w_\alpha^{ui}}{w_\beta^{ui}} = \frac{y_\beta q_\beta + y_\alpha q_\alpha}{y_\beta(q_\alpha+q_\beta)}, \tag{8}$$

where $(\overline{\gamma}-a)(q_\alpha y_\alpha + y_\beta q_\beta) < w_\alpha^{ui} \leq w_\alpha^*$ and $(\overline{\gamma}-a)(q_\alpha + q_\beta)y_\beta < w_\beta^{ui} \leq \frac{w_\alpha^* y_\beta(q_\alpha+q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)}$, is an equilibrium price pair in this configuration. In addition, when $w_\alpha^{ui} = w_\alpha^*$ any price combination such that,

$$w_\alpha^{ui} = w_\alpha^* \tag{9}$$

$$w_\beta^{ui} \geq \frac{w_\alpha^* y_\beta(q_\alpha+q_\beta)}{(y_\beta q_\beta + y_\alpha q_\alpha)}, \tag{10}$$

is an equilibrium price too. It remains to specify the necessary condition for this configuration to occur. From condition (6), configuration $C_I$ occurs only if $(\overline{\gamma}-a)(y_\alpha q_\alpha + y_\beta q_\beta) - w_\alpha^{ui} < 0$. This results in the following necessary condition,

$$\frac{\overline{\gamma}}{a} < \frac{4f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}. \tag{11}$$

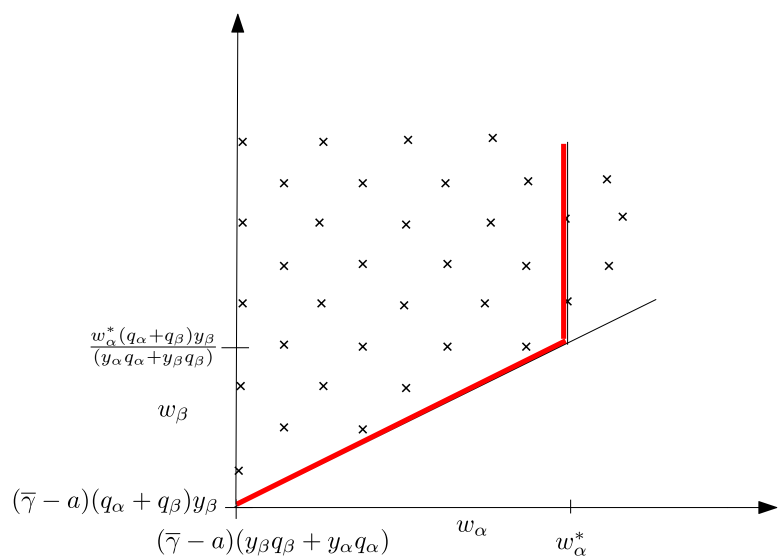


**Figure 7** **The reaction correspondence of platform $\beta$ given $w_\alpha > (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha)$ and the reaction curve of platform $\alpha$ given $w_\beta > (\overline{\gamma}-a)$. Their intersection points give the equilibrium price pairs in this market configuration. These are depicted by the thick line.**

**Uncovered Market - $C_{II}$.** In this case we suppose *ex ante* that the market is uncovered with both platforms serving the market. We first identify the equilibrium prices and then the values of $(\overline{\gamma}, a)$ for which this market configuration is feasible. The condition for an uncovered market in which both the high-quality and low-quality platforms serve the market is given by,

$$\overline{\gamma} - a < \frac{w_\beta}{(q_\alpha+q_\beta)y_\beta} < \frac{(w_\alpha - w_\beta)}{q_\alpha(y_\alpha - y_\beta)} < \overline{\gamma} + a. \tag{12}$$

The best reply functions of the respective platforms are obtained from the first order conditions of the platforms profit functions and are given below.

$$w_\alpha(w_\beta) = \frac{5f}{9}(y_\alpha - y_\beta) + \frac{f}{9}\overline{\gamma}(y_\alpha - y_\beta) + \frac{1}{2}w_\beta, \tag{13}$$

$$w_\beta(w_\alpha) = \frac{1}{6}\frac{y_\beta(f(\overline{\gamma}-a)(y_\alpha - y_\beta) + 9w_\alpha)}{(2y_\alpha + y_\beta)}. \tag{14}$$

Note that the above functions are linear. Solving the above two simultaneous equations yields the following unique equilibrium prices,

$$w_\alpha^u = \frac{f((8\overline{\gamma}+40a)y_\alpha^2 - (23\overline{\gamma}+a)y_\beta y_\alpha - (17a+7\overline{\gamma})y_\beta^2)}{9(y_\beta+8y_\alpha)}, \tag{15}$$

$$w_\beta^u = \frac{4fy_\beta(\overline{\gamma}+2a)(y_\alpha - y_\beta)}{3(y_\beta+8y_\alpha)}. \tag{16}$$

The reaction functions and their intersection point are shown in Figure 8. Since the market is not covered the lowest quality content provider does not join the lower quality platform. Therefore a necessary condition for the above configuration to hold is $(\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta - w_\beta^u < 0$. Substituting for $w_\beta^u$ the above condition can be rewritten as,

$$\frac{\overline{\gamma}}{a} < \frac{2f(y_\alpha - y_\beta)+30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}. \tag{17}$$

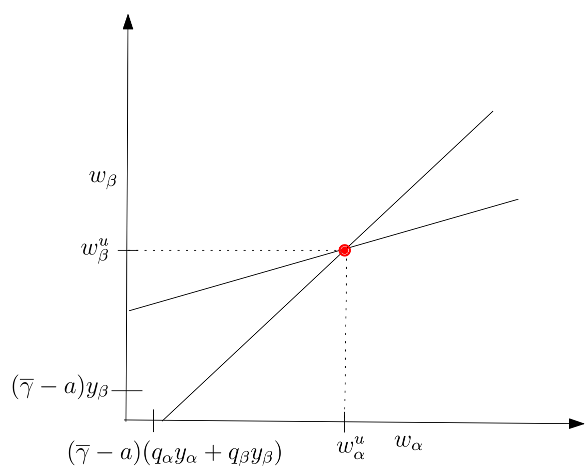


**Figure 8** **The reaction curve of platform $\beta$ given $w_\alpha > (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha)$ and the reaction curve of platform $\alpha$ given $w_\beta > (q_\alpha+q_\beta)(\overline{\gamma}-a)y_\beta$ intersect at the price pair $(w_\beta^u, w_\alpha^u)$.**

**Covered market- $C_{III}$.** We now suppose *ex ante* that the market is covered with both platforms serving the market. We again identify the equilibrium prices and then the values of $(\overline{\gamma}, a)$ for which this market configuration is feasible. Proceeding as we did in the previous market configurations, to derive the equilibrium prices, we first derive best response prices of each platform to the price set by the other platform. The condition for a covered market in which both the high-quality and low-quality platforms serve the market is

$$\frac{w_\beta}{(q_\alpha+q_\beta)y_\beta} \le \overline{\gamma} - a < \frac{(w_\alpha - w_\beta)}{q_\alpha(y_\alpha - y_\beta)} < \overline{\gamma}+a. \tag{18}$$

The first order conditions associated with the profit functions for both platforms yield the following best reply functions,

$$w_\alpha(w_\beta) = (y_\alpha - y_\beta)(\frac{1}{3}f(\overline{\gamma}+a) - \frac{2}{9}f^2(\overline{\gamma}-a)) + \frac{1}{2}w_\beta,$$
$$w_\beta(w_\alpha) = \begin{cases} (y_\alpha - y_\beta)(\frac{1}{3}f(a-\overline{\gamma}) - \frac{1}{18}f^2(\overline{\gamma}-a)) + \frac{1}{2}w_\alpha & \text{if } w_\alpha < w_\alpha^* \\ (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta & \text{if } w_\alpha \ge w_\alpha^* \end{cases}$$

where $w_\alpha^* = y_\alpha(\frac{2}{9}f^2(a-\overline{\gamma}) + \frac{1}{3}f(\overline{\gamma}+a)) - y_\beta(\frac{2}{9}f^2(a-\overline{\gamma}) + \frac{1}{6}f(5a-\overline{\gamma}))$.

*Interior Solution.* From the above best response functions we get the following unique equilibrium prices in the case of an interior solution.

$$w_\alpha^{ci} = (y_\alpha - y_\beta)\frac{1}{27}f(7f(\overline{\gamma} - a) - 6(3a - \overline{\gamma})), \tag{19}$$

$$w_\beta^{ci} = (y_\alpha - y_\beta)\frac{2}{27}f(f(\overline{\gamma} - a) + 3(3a - \overline{\gamma})). \tag{20}$$

A market is covered with an interior solution in the price subgame if the price charged by the lower quality platform is lower than the value derived by the lowest quality content provider, i.e., $(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta - w_\beta^{ci} > 0$. In this market configuration the lowest quality content provider prefers the lowest quality platform, otherwise we have a preempted market. Moreover, the lowest-quality content provider's net utility must also be positive. Thus the following condition has to hold in equilibrium,

$$(\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta - w_\beta^{ci} > \max\{w_\alpha^{ci} - (\overline{\gamma} - a)(y_\beta q_\beta + y_\alpha q_\alpha), 0\}.$$

By plugging the equilibrium prices in (19) and (20) into the above inequality, we obtain the following necessary conditions on the tuple $(y_\alpha, y_\beta, \overline{\gamma}, a)$ for this configuration to exist.

$$\frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta} < \overline{\gamma} < \frac{5f + 18}{5f + 6}. \tag{21}$$

*Corner solution.* We denote the content provider market to be covered with a corner solution in the price

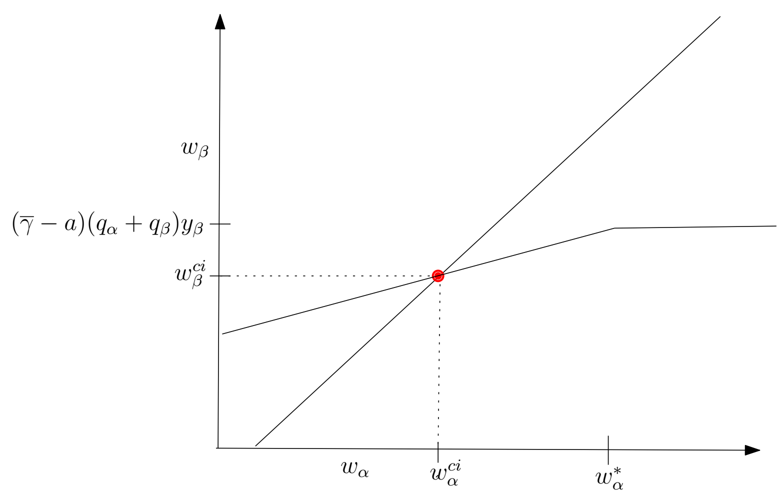


**Figure 9 The best response functions of platform $\beta$ and $\alpha$ given $\frac{2f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha - y_\beta)+6y_\alpha+21y_\beta)} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$. These curves intersect at $(w_\beta^{ci}, w_\alpha^{ci})$.**

subgame if the lower quality platform quotes a price that is just sufficient so that the lowest quality content provider joins the platform. In this case, a corner solution occurs and we have the following price charged by platform $\beta$,

$$w_\beta^{cc} = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta. \tag{22}$$

From the first order conditions of the high quality profit function we deduce that the equilibrium price is given by,

$$w_\alpha^{cc} = \frac{1}{18}(f4f(a - \overline{\gamma}) + 6(a - \overline{\gamma}))y_\alpha + \frac{1}{18}(4f(\overline{\gamma} - a + 3(\overline{\gamma} - 5a))y_\beta. \tag{23}$$

For configuration $C_{III}$ to occur with a corner solution the following three conditions need to hold,

$$(\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta - w_\beta^{cc} > (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha) - w_\alpha^{cc},$$

$$w_\beta^{ci} \geq (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta, \tag{24}$$

$$w_\beta^{u} \leq (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta. \tag{25}$$

The above inequalities yield the following necessary and sufficient conditions on $\overline{\gamma}, y_\alpha, y_\beta$ for the above equilibrium prices to yield Configuration $C_{III}$,

$$\frac{2f(y_\alpha-y_\beta)+30y_\alpha-3y_\beta}{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta} \leq \frac{\overline{\gamma}}{a} \leq \min\left\{\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}\right\}. \tag{26}$$

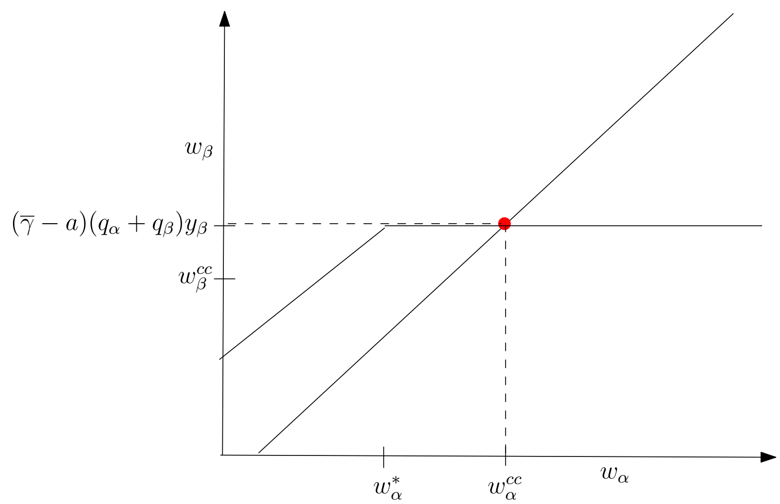


**Figure 10 The best response functions of platform $\beta$ and $\alpha$ given $\frac{2f(y_\alpha-y_\beta)+30y_\alpha-3y_\beta}{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta} \leq \frac{\overline{\gamma}}{a} \leq \min\left\{\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}\right\}$. These curves intersect at $(w_\beta^{cc}, w_\alpha^{cc})$.**

**Covered Preempted market $C_{IV}$.**

In this case we suppose *ex ante* that the market is covered with only the high quality platform serving the market. We identify the equilibrium prices for this market configuration and derive the best price responses of each platform in the usual way. The condition for a covered market where only the high-quality platform participates in the market is,

$$(\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha) - w_\alpha \geq \max\{0, (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta - w_\beta\}. \tag{27}$$

The profit functions for platforms $\alpha$ and $\beta$ given $y_\alpha$ and $y_\beta$ are

$$\pi_\alpha^p = \frac{8}{9}f^2\overline{\gamma}(y_\alpha-y_\beta)+w_\alpha, \tag{28}$$

$$\pi_\beta^p = \frac{2}{9}f^2\overline{(}y_\alpha-y_\beta). \tag{29}$$

We note that in this configuration, given $w_\alpha$, platform $\beta$'s profit does not depend on $w_\alpha$. Thus any $w_\beta$ that meets the condition specified by (27) is a best response. Given $w_\beta$, it follows from the condition specified by (27) and the first order conditions of (28), that

$$w_\alpha = \begin{cases} (\overline{\gamma}-a)(q_\alpha y_\alpha + q_\beta y_\beta) & \text{if } w_\beta \geq (\overline{\gamma}-a)(q_\alpha + q_\beta)y_\beta, \\ (\overline{\gamma}-a)q_\alpha(y_\alpha - y_\beta) + w_\beta & \text{if } w_\beta < (\overline{\gamma}-a)(q_\alpha + q_\beta)y_\beta. \end{cases} \quad (30)$$

Thus the above characterizes the price equilibrium combinations for configuration $C_{IV}$.

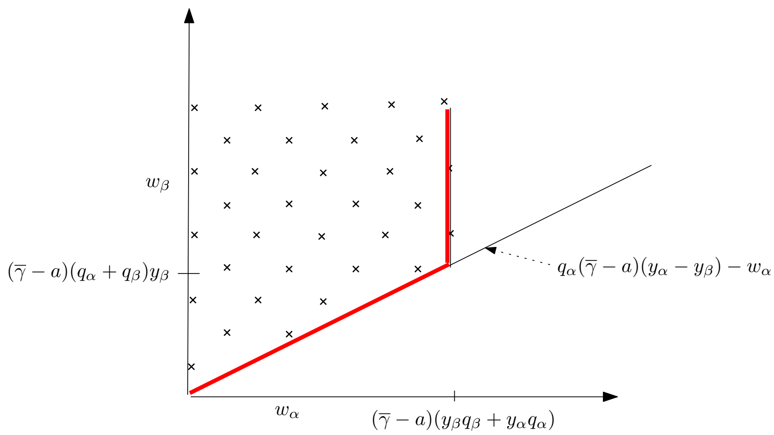


**Figure 11 The reaction correspondence of platform $\beta$ given $w_\alpha \leq (\overline{\gamma}-a)(y_\beta q_\beta + y_\alpha q_\alpha)$ and the reaction curve of platform $\alpha$ given $w_\beta$. Their intersection points give the equilibrium price pairs in this market configuration. These are depicted by the thick line.**

### A.3. Nash Equilibrium in the Price Subgame

In this section, we show the existence of pure strategy Nash equilibrium in the price-subgame. We look at the equilibrium price pairs derived in the pervious section and determine if they are best replies across all the configurations. We characterize the price subgame equilibria in terms of the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ and give the conditions for their existence. Specifically, we give the conditions for these price equilibria to yield their corresponding market configurations.

We show that the uncovered market configuration, $(C_I)$, does not occur at a subgame price equilibrium. We then show that market configurations $C_{II}$, $C_{III}$ and $C_{IV}$ exist. In doing so, we determine the set of parametric values $(\overline{\gamma}, y_\alpha, y_\beta, a)$ for which these different configurations exist and characterize the prices in each configuration using the same parameters.

In the remaining part of this section, we show that configuration $C_I$ does not exist while $C_{II}$, $C_{III}$ and $C_{IV}$ exist. We also give the conditions for their existence given the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ and their accompanying equilibrium prices.

Lemma 2. *Any equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui}) \in \mathcal{R}_I$ is not a pure strategy Nash equilibrium in the price subgame.*

*Proof.* We assume to arrive at a contradiction that their exists a pair of equilibrium prices $(w_\alpha^{ui}, w_\beta^{ui}) \in \mathcal{R}_I$ that are a pure strategy Nash equilibrium in the price subgame. To prove our Lemma we show that given a subgame $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that condition in (11) is met, the prices in the pair $(w_\beta^{ui}, w_\alpha^{ui})$ are not best reply pairs on the whole domain of strategies, i.e, there exists for at least one platform the incentive to deviate to a price that will yield a different configuration and higher profits. In particular, we show that $w_\beta^{ui}$ does not beat all price strategies in the projection of $\mathcal{R}_{II} \bigcup \mathcal{R}_{III} \bigcup \mathcal{R}_{IV}$ against $w_\alpha^{ui}$.

As shown in section A.2 there are two possible characterizations for the equilibrium price pair that holds if configuration $C_I$ is exogenously imposed. We show that prices satisfying both characterizations are not best reply pairs on the whole domain.

*Case I. Equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui})$ in (9) and (10).*

As previously discussed in section A.2, the above price characterizations yield configuration $C_I$ only if the condition in (11) is met. We denote the profit for platform $\beta$ under the price pair $(w_\alpha^{ui}, w_\beta^{ui})$ as $\pi_\beta^{ui}$ and that under the pair $(w_\alpha^{ui}, \overline{w}_\beta)$ as $\overline{\pi}_\beta$. We also denote the difference between the two profits, $\pi_\beta^{ui} - \overline{\pi}_\beta$, as $d(\overline{\gamma})$. Let $\overline{\gamma}^*$ denote the upper bound value of $\gamma$ such that configuration $C_I$ is possible. We now show that there exists configurations with price pairs $(w_\alpha^{ui}, \overline{w}_\beta)$ such that $\pi_\beta^u < \overline{\pi}_\beta$ for $\overline{\gamma} < \overline{\gamma}^*$, which implies that these price characterization cannot be a subgame equilibrium. For this purpose we fix $w_\alpha^{ui}$ and consider profits of platform $\beta$ under configurations $C_{III}$ and $C_{IV}$.

Let $\overline{w_\beta} = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$, then configuration $C_{III}$ will arise whenever $\overline{\gamma}^{**} < \overline{\gamma} < \overline{\gamma}^*$, where $\overline{\gamma}^{**} = (a(4f(y_\alpha - y_\beta) + 33y_\beta - 6y_\alpha))/(4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta)$. The function $d(\overline{\gamma})$ is convex since $\partial^2 d(\overline{\gamma})/\partial^2(\overline{\gamma}) > 0$. Moreover, $d(\overline{\gamma})$ has two roots:

$$\overline{\gamma}_1 = a\frac{4f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}, \quad \text{and} \quad \overline{\gamma}_2 = a.$$

It follows that whenever $\overline{\gamma}_2 \leq \overline{\gamma} < \overline{\gamma}^*$ then $d(\overline{\gamma}) < 0$. This implies that platform $\beta$ would prefer to deviate to a covered market with a corner solution. However, this configuration is possible for all values of $\overline{\gamma} > a$ when $\overline{\gamma}^{**} < \overline{\gamma}_2$. And this occurs when $y_\alpha/y_\beta \geq 5/2$. So we now proceed to show that when $y_\alpha/y_\beta < 5/2$ that platform $\beta$ would prefer to deviate to configuration $C_{IV}$ where all masses of CPs join it.

Let $\overline{w_\beta} = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ and consider the case when $y_\alpha/y_\beta < 5/2$. It follows that $\overline{\gamma}^{**} > a$, so for $a < \overline{\gamma} \leq \overline{\gamma}^{**}$ a covered preempted market results. The difference $d(\overline{\gamma})$ under this configuration is convex since $\partial^2 d(\overline{\gamma})/\partial^2 \overline{\gamma} > 0$. The roots of $d(\overline{\gamma})$ are $r1$ [3] and $a$. Moreover, $r1 > \overline{\gamma}^{**}$ whenever $1 < y_\alpha/y_\beta < 9/f + 1$. Since $5/2 \leq 9/f + 1$ for $f \in (0,1]$) platform $\beta$ prefers to deviate to configuration $C_{IV}$ whenever $y_\alpha/y_\beta < 5/2$.

*Case II. Equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui})$ in (8).*

We show that if platform $\beta$ picks the price $\overline{w}_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ then it makes a higher profit in the resulting configuration. We denote the profit of platform $\beta$ under the price pair $(w_\alpha^{ui}, w_\beta^{ui})$ as $\pi_\beta^{ui}$ and that under the price pair $(w_\alpha^{ui}, \overline{w}_\beta)$ as $\overline{\pi}_\beta$. Note that when platform $\beta$ picks the price $\overline{w_\beta}$ the market becomes covered and configuration $C_{III}$ emerges. The function $\overline{\pi}_\alpha - \pi_\alpha = d(w_\alpha^{ui})$ is increasing in $w_\alpha^{ui}$ since $\partial d(w_\alpha^{ui})/\partial w_\alpha^{ui} > 0$. Moreover, it has a single root at $w_\alpha^{ui*} = (\overline{\gamma} - a)(y_\alpha q_\alpha + y_\beta q_\beta)$. Thus for given any $w_\alpha^{ui} > w_\alpha^{ui*}$ platform $\beta$ would

[3] The root $r1$ can be expressed as $a(4y_\beta^2 f^2 + 4f^2 y_\alpha^2 + 3fy_\alpha y_\beta + 216y_\alpha y_\beta - 8f^2 y_\alpha y_\beta + 108y_\beta^2 + 3fy_\beta^2 - 6fy_\alpha^2)/f/(4fy_\alpha^2 + 6y_\alpha^2 - 3y_\alpha y_\beta - 8fy_\alpha y_\beta + 4fy_\beta^2 - 3y_\beta^2)$.

prefer to deviate to price $\overline{w}_\beta$. Therefore an equilibrium price pair $(w_\alpha^{ui}, w_\beta^{ui})$ for which the characterization in (10) holds is not a subgame Nash equilibrium. □

We now show that the equilibrium price pair characterized for configuration $C_{II}$ in Section A.2 is a subgame equilibrium price pair and give the conditions on $\overline{\gamma}$, $a\, y_\beta$, and $y_\alpha$ for this to hold.

LEMMA 3. *Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, there exists a unique equilibrium price pair $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{II}$ only if*

$$1 < \frac{\overline{\gamma}}{a} < \frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}.$$

*Proof.* From section A.2 we know that the prices in the pair $(w_\alpha^u, w_\beta^u)$ are unique and mutual best replies in the restricted domain $\mathcal{R}_{II}$; which corresponds to the market configuration $C_{II}$. Therefore this price pair is our only candidate for the price equilibrium that falls in $\mathcal{R}_{II}$. To show that the candidate pair $(w_\alpha^u, w_\beta^u)$ is a price subgame equilibrium, we need to show that the prices in these pair are also mutual best replies on the whole domain of strategies, i.e, given price $w_\alpha^u$, platform $\beta$ does not have an incentive to change to price $\overline{w}_\beta$ which will result in another configuration and a higher profit. Formally, we have to show that $w_\beta^u$ beats any strategy $w_\beta$ in the projection $R_I \cup R_{III} \cup R_{IV}$ against $w_\alpha^u$ and vice versa. We denote the equilibrium price candidate $(w_\alpha^u, w_\beta^u)$ as $(w_\alpha^*, w_\beta^*)$.

We first fix $w_\beta^*$ and show that platform $\alpha$ has no incentive to deviate to any price $\overline{w}_\alpha$ in any configuration. We denote the profit under the price pair $(w_\alpha^*, w_\beta^*)$ as $\pi_\alpha^*$ and that under the pair $(\overline{w}_\alpha, w_\beta^*)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi_\alpha^* - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

1. *Platform $\alpha$ has no incentive to deviate to configuration $C_I$.* We find platform $\alpha's$ best reply given $w_\beta^*$ under market configuration $C_I$ and show that the profit realized is less than that under configuration $C_{II}$ at price $w_\alpha^*$. Let $\overline{w}_\alpha$ be the best reply of platform under configuration $C_I$. It is given as the solution to the following maximization problem,

$$\begin{aligned} &\max\ \pi_\alpha(w_\alpha, w_\beta^*),\\ &\text{s.t. } w_\alpha \le \frac{w_\beta^*}{y_\beta}\frac{q_\alpha y_\alpha + q_\beta y_\beta}{q_\alpha + q_\beta}. \end{aligned}$$

The constraint in the above problem arises from the necessary conditions expressed in (6) for market configuration $C_I$ to hold. The profit function $\pi_\alpha$ is concave in $w_\alpha$ since $\partial^2\pi_\alpha(w_\alpha)/\partial^2 w_\alpha < 0$. The unconstrained optimal solution to the above maximization problem is lager than the constraint. Therefore the constraint binds and it is the best reply.

We now compare the two profits under both configurations. After evaluating the difference $d(\overline{\gamma}) = \pi_\alpha^* - \overline{\pi}_\alpha$, we obtain that $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because, $\partial^2 d(\overline{\gamma})/\partial^2\overline{\gamma} > 0$. In addition, $d(\overline{\gamma}) \ge 0$ since $d(\overline{\gamma})$ is a quadratic function in $\overline{\gamma}$ with a single root at $(a4(1+f)y_\alpha + (2+f)y_\beta)/(4(f-1)y_\alpha + (f-2)y_\beta)$. Therefore, given $w_\beta^*$ platform $\alpha$ has no incentive to deviate to a price $\overline{w}_\alpha$ that would result in configuration $C_I$.

2. *Platform $\alpha$ cannot deviate to configuration $C_{III}$.* Form section A.2 we know that $w_\beta^*$ is defined only if the condition in (17) is satisfied. This implies that $w_\beta^* > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. Therefore, it is not possible to have a covered market with content providers patronizing the two platforms when platform $\beta's$ price is fixed at $w_\beta^*$.

3. *Platform $\alpha$ has no incentive to deviate to configuration $C_{IV}$.* We proceed in a similar manner to the first case. We find platform $\alpha's$ best reply given $w^*_\beta$ under market configuration $C_{IV}$ and show that the profit realized is less than that under configuration $C_{II}$ at price $w^*_\alpha$. Let $\overline{w}_\alpha$ be the best reply of platform under configuration $C_{IV}$. It is given as the solution to the following maximization problem

$$\max\ \pi_\alpha(w_\alpha, w^*_\beta),$$
$$\text{s.t. } w_\alpha \leq (\overline{\gamma} - a)q_\alpha y_\alpha + q_\beta y_\beta$$

Since $\pi_\alpha$ is linear and increasing in $w_\alpha$, the constraint binds and is the best response. Under this price $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because $\partial^2 d(\overline{\gamma})/\partial^2\overline{\gamma} > 0$. The function $d(\overline{\gamma})$ has two roots $\overline{\gamma}_1$ and $\overline{\gamma}_2$, these have been defined in the proof of the previous Lemma. Since configuration $C_{II}$ is defined outside these two roots it follows that $d(\overline{\gamma})$ is positive. Therefore, platform $\alpha$ has no incentive to deviate to configuration $C_{IV}$.

We now fix $w^*_\alpha$ and show that platform $\beta$ has no incentive to deviate to any price $\overline{w}_\beta$ that will yield another configuration. We denote the profit of platform $\beta$ under the price pair $(w^*_\alpha, w^*_\beta)$ as $\pi^*_\beta$ and that under the pair $(\overline{w}_\beta, w^*_\alpha)$ as $\overline{\pi}_\beta$. We denote the difference $\pi^*_\beta - \overline{\pi}_\beta$ by $d(\overline{\gamma})$.

1. *Platform $\beta$ has no incentive to deviate to configuration $C_I$.* We proceed in a similar fashion to the previous parts. We find platform $\beta's$ best reply given $w^*_\alpha$ under market configuration $C_I$ and show that the profit realized is less than that under configuration $C_{II}$ at price $w^*_\beta$. Let $\overline{w}_\beta$ be the best reply of platform under configuration $C_I$. It is given as the solution to the following maximization problem,

$$\max\ \pi_\beta(w^*_\alpha, w_\beta),$$
$$\text{s.t. } w_\beta \geq \frac{w^*_\alpha y_\beta(q_\alpha + q_\beta)}{q_\alpha y_\alpha + q_\beta y_\beta}.$$

The constraint in the above problem arises from the necessary conditions expressed in (6) for market configuration $C_I$ to hold. The profit function $\pi_\beta$ is concave in $w_\beta$ since the $\frac{\partial^2 \pi_\beta(w_\beta)}{\partial^2 w_\beta} > 0$. Through computation one can show that the optimal solution is at the boundary since the constraint binds.
We now compare the two profits. The difference, $d(\overline{\gamma})$, is a convex function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. Moreover, this function is a quadratic function in $\overline{\gamma}$ with a single root at $\gamma = \frac{a(4y_\alpha + 4fy_\alpha + 2y_\beta + fy_\beta}{4fy_\alpha + fy_\beta - 4y_\alpha - 2y_\beta)}$. Therefore, for all $\overline{\gamma}$ the difference $d(\overline{\gamma}) \geq 0$. Thus given $w^*_\alpha$, platform $\beta$ has no incentive to deviate to a price that results in $C_I$.

2. *Platform $\beta$ has no incentive to deviate to configuration $C_{III}$.* We show that platform $\beta$ makes more profit under configuration $C_{II}$ than if it changed its price and deviated to configuration $C_{III}$. Let $\overline{w}_\beta$ be the best response price under configuration $C_{III}$ given $w^*_\alpha$. It is defined below,

$$\overline{w}_\beta = \text{argmax}\ \pi_\beta(w^*_\alpha, w_\beta),$$
$$\text{s.t. } w_\beta \leq (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta. \tag{31}$$

The above profit function is concave in $w_\beta$ since $\frac{\partial^2 \pi_\beta}{\partial^2 w_\beta} < 0$. Moreover one can show through computation that the constraint in problem (31) binds at the optimum. We now compare profits under $C_{II}$ and those resulting in $C_{III}$ under the deviation price $\overline{w}_\beta$. The difference in profits given by $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. In addition, the function $d(\overline{\gamma})$ has a single root at $\overline{\gamma} = a\frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}$ therefore $d(\overline{\gamma}) \geq 0$. Thus given $w^*_\alpha$, platform $\beta$ has no incentive to deviate to a price that results in configuration $C_{III}$
.

3. *Platform $\beta$ has no incentive to deviate to configuration $C_{IV}$.* If platform $\beta$ chooses to deviate to a configuration where all CPs subscribe to it, the best price it can offer is given by $\overline{w}_\beta = w^*_\alpha + (k+a)q_\alpha(y_\beta - y_\alpha)$. Platform $\beta$ has no incentive to deviate in this case. Therefore we consider cases where $\overline{w}_\beta$ is positive. Proceeding in a similar manner to the previous cases we can show that $d(\overline{\gamma}) \geq 0$ for $\overline{\gamma} > a$. This implies that platform $\beta$ has no incentive to deviate to configuration $C_{IV}$.

□

We have shown that the equilibrium price pair $(w^u_\alpha, w^u_\beta)$ for which condition (17) holds is a pure strategy Nash Equilibrium in the price subgame. We next show that configuration $C_{III}$ with a corner solution exists and give both the necessary and sufficient conditions under which this configuration exists.

LEMMA 4. *Let Assumption 1 hold. Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, there exists a unique equilibrium price pair $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{III}$ such that $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ only if*

$$\frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta} \leq \frac{\overline{\gamma}}{a} \leq \min\left\{\frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta}, \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}\right\}.$$

*Proof.* In Section A.2 we determined that the prices, in the unique equilibrium pair $(w^{cc}_\alpha, w^{cc}_\beta)$, are unique and mutual best replies in the restricted domain $\mathcal{R}_{III}$ if a covered market configuration with a corner solution[4] was assumed. Thus this price pair is our only price subgame equilibrium candidate.

Since our only candidate pair is $(w^{cc}_\alpha, w^{cc}_\beta)$, we need to show that the prices in these pair are also mutual best replies on the whole domain of strategies, i.e, given price $w^{cc}_\alpha$, platform $\beta$ does not have an incentive to change to price $\overline{w_\beta}$ which will result in another configuration and a higher profit. We show that $w^{cc}_\beta$ beats any strategy $\overline{w_\beta}$ in the projection $R_I \cup R_{II} \cup R_{IV}$ against $w^{cc}_\alpha$ and vice versa. We denote $w^*_\alpha = w^{cc}_\alpha$ and $w^*_\beta = w^{cc}_\beta$.

We first fix $w^*_\beta$ and show that given this price, platform $\alpha$ has no incentive to deviate to a price that would result in configuration $C_I$, $C_{II}$ or $C_{IV}$. We note that under the price $w^*_\beta$ it is not possible to have the uncovered configurations $C_I$ or $C_{II}$ since all content providers have an incentive to participate. So we only look at the possibility of deviating to configuration $C_{IV}$. We denote the profit under the price pair $(w^*_\alpha, w^*_\beta)$ as $\pi^*_\alpha$ and that under the pair $(\overline{w}_\alpha, w^*_\beta)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

1. *Platform $\alpha$ has no incentive to deviate to configuration $C_{IV}$.* We find platform $\alpha's$ best reply given $w^*_\beta$ under market configuration $C_{IV}$ and show that the profit realized is less than that under configuration $C_{III}$ at price $w^*_\alpha$. Let $\overline{w}_\alpha$ be the best reply of platform under configuration $C_{IV}$. It is given by,

$$\overline{w}_\alpha = \text{argmax } \pi_\alpha(w_\alpha, w^*_\beta),$$
$$\text{s.t. } w_\alpha \leq (\overline{\gamma} - a)q_\alpha y_\alpha + q_\beta y_\beta.$$

The constraint in the above problem arises from the necessary condition expressed in (27) for market configuration $C_{IV}$ to hold. The profit function $\pi_\alpha$ is linear and increasing in $w_\alpha$. Therefore the constraint binds and it is the best reply. We now compare profit at the price pair $(w^*_\alpha, w^*_\beta)$ in configuration $C_{III}$ to that under the pair $(\overline{w}_\alpha, w^*_\beta)$ in configuration $C_{IV}$. The difference in profits is given by $d(\overline{\gamma})$ which is a convex function in $\overline{\gamma}$ because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[5] Moreover, $d(\overline{\gamma})$ has a single root at $\gamma = \frac{a(4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta)}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}$, therefore $d(\overline{\gamma}) \geq 0$. Consequently platform $\alpha$ has no incentive to deviate to configuration $C_{IV}$.

[4] A corner solution refers to the instance when $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.

[5] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > (216a(y_\alpha - y_\beta))^{-1}(-(y_\alpha - y_\beta)^2 16f^3 + (y_\alpha - y_\beta)^2 32f^2 + (12y_\alpha y_\beta + 84y^2_\alpha - 15y^2_\beta)f)$.

We now fix $w_\alpha^*$ and show that platform $\beta$ has no incentive to deviate to any price $\overline{w}_\beta$. We note that it is not possible for platform $\beta$ to come up with prices which will result in configuration $C_{IV}$ where all CP's flock to platform $\alpha$, because $w_\alpha^*$ is defined only for $\overline{\gamma} \leq \min\left\{\frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta}, \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}\right\}$, where as configuration $C_{IV}$ results only if $\overline{\gamma} > \frac{2f(y_\alpha-y_\beta)+18y_\alpha+9y_\beta}{2f(y_\alpha-y_\beta)+6y_\alpha+21y_\beta}$. We denote the profit of platform $\beta$ under the price pair $(w_\alpha^*, w_\beta^*)$ as $\pi_\beta^*$ and that under the pair $(\overline{w}_\beta, w_\alpha^*)$ as $\overline{\pi}_\beta$. We denote the difference $\pi_\beta^* - \overline{\pi}_\beta$ as $d(\overline{\gamma})$.

1. *Platform $\beta$ has no incentive to deviate to configuration $C_I$* We show that the best response given $w_\alpha^*$, such that configuration $C_I$ emerges, will yield a lower profit. Let $\overline{w}_\beta$ denote the best response under $C_I$ given $w_\alpha^*$. It is given by,

$$\begin{aligned}\overline{w}_\beta =& \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),\\ &\text{s.t. } w_\beta \geq \frac{w_\alpha^*(q_\alpha+q_\beta)y_\beta}{q_\beta y_\beta + q_\alpha y_\alpha}.\end{aligned}$$

For this configuration to occur we need the condition in (6) to be satisfied hence the constraint in the above maximization problem. Since $\pi_\beta$ is independent of $w_\beta$ we have the best response satisfying the constraint inequality i.e., $\overline{w}_\beta \geq \frac{w_\alpha^*(q_\alpha+q_\beta)y_\beta}{q_\beta y_\beta+q_\alpha y_\alpha}$. The function $d(\overline{\gamma})$ is a concave function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[6] Moreover, $d(\overline{\gamma})$ has two roots at

$$\gamma_1 = a \quad \text{and} \quad \gamma_2 = \frac{a(4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta)}{4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta}.$$

Thus for all $\gamma_1 \leq \overline{\gamma} \leq \gamma_2$, we have $d(\overline{\gamma}) \geq 0$. In Section A.2 the equilibrium pair $(w_\alpha^{cc}, w_\beta^{cc})$ is defined only if $\overline{\gamma} \in [\gamma_1, \gamma_2]$. Therefore platform $\beta$ has no incentive to deviate to a price that results in $C_I$.

2. *Platform $\beta$ has no incentive to deviate to configuration $C_{II}$*. This follows from the fact that the maximization problem given below has no solution.

$$\begin{aligned}\overline{w}_\beta =& \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),\\ &\text{s.t. } w_\beta > (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta.\end{aligned}$$

We note that the supremum to the this problem is given by $\overline{w}_\beta = (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta$. Therefore any price $w_\beta$ satisfying the maximization constraint will yield a lower profit.

3. *Platform $\beta$ has no incentive to deviate to configuration $C_{IV}$*. If platform $\beta$ chooses to deviate to a configuration where all CPs subscribe to it, the best price it can offer is denoted by $\overline{w}_\beta$ and is given by,

$$\begin{aligned}\overline{w}_\beta =& \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),\\ &\text{s.t. } w_\beta \leq w_\alpha^* + (\overline{\gamma}+a)q_\alpha(y_\beta - y_\alpha).\end{aligned}$$

The profit function is increasing in $w_\beta$, therefore the constraint binds and we have $\overline{w}_\beta = w_\alpha^* + (k+a)q_\alpha(y_\beta - y_\alpha)$.[7] The difference $d(\overline{\gamma})$ between the profits under the price pair $(w_\alpha^*, w_\beta^*)$ in configuration $C_{III}$ and that

[6] $\partial^2 d(\overline{\gamma})/\partial^2(\overline{\gamma}) < \partial^2 d(\overline{\gamma})/\partial^2(\overline{\gamma}) < (f(-12y_\alpha^3 - 108y_b^2 y_\alpha - 33y_\beta^3 - 90y_\beta y_\alpha^2 - 48fy_\alpha^2 y_\beta + 3fy_\alpha y_\beta^2 + 4fy_\alpha^3 + 41fy_\beta^3 + 8y_\alpha^3 f^2 + 4y_\beta^3 f^2 - 12y_\alpha^2 f^2 y_\beta))/(108a(y_\alpha - y_\beta)(2y_\alpha + y_\beta))$.

[7] The constraint directly arises from the utility maximization by the CPs. In particular, all CPs have to prefer joining the low quality platforms including those with highest quality $(\overline{\gamma}+a)$.

under price pair $(w^*_\alpha, \overline{w}_\beta)$ in configuration $C_{IV}$ is concave whenever $y_\alpha \leq y_\beta \frac{9+f}{f}$ and convex vice versa.[8] Moreover, $d(\overline{\gamma})$ has two roots at

$$\gamma_1 = \frac{a((3-f)y_\beta + (-12+f)y_\alpha)}{((-9-f)y_\beta + f y_\alpha)} \quad \text{and} \quad \gamma_2 = \frac{a((15-4f)y_\beta + (-6+4f)y_\alpha)}{((-4f+3)y_\beta + (6+4f)y_\alpha)}. \tag{32}$$

One can show that when $y_\alpha \leq y_\beta \frac{9+f}{f}$ the interval in which configuration $C_{III}$ is defined lies between the interval defined by the two roots. Since in this case $d(\overline{\gamma})$ is concave the difference is positive implying that platform $\beta$ has no incentive to deviate. In the case where For the region in which configuration $C_{III}$ is defined $d(\overline{\gamma}) > 0$ since previous cases we can show that $d(\overline{\gamma}) \geq 0$ for $\overline{\gamma} > a$. This implies that platform $\beta$ has no incentive to deviate. In the case when $y_\alpha \geq y_\beta \frac{9+f}{f}$ the roots given by (32) above are negative. Since $d(\overline{\gamma})$ is convex and configuration $C_{III}$ is defined only for positive $\overline{\gamma}$ we have that platform $\beta$ has no incentive to deviate.

□

We now show that configuration $C_{III}$ with an interior solution exists and give both the necessary conditions under which this configuration exists.

Lemma 5. *Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, there exists a unique equilibrium price pair $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{III}$ such that $w^*_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ only if*

$$\frac{2f(y_\alpha - y_\beta) + 9y_\beta + 18y_\alpha}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}.$$

*Proof.* We follow the same line of proof applied in the previous two lemmas. From section A.2, we know that the prices in the pair $(w^{ci}_\alpha, w^{ci}_\beta)$ are unique and mutual best replies in the restricted domain $\mathcal{R}_{III}$; if a covered market configuration was assumed and an interior solution resulted.[9] Thus this price pair is our only candidate for the price equilibrium pair that falls in $\mathcal{R}_{III}$ (with an interior solution). Moreover, it is also shown in the same section that for $(w^{ci}_\alpha, w^{ci}_\beta)$ to be in $\mathcal{R}_{III}$ it is necessary and sufficient that the condition expressed in (21) holds.

We now show that the prices in the equilibrium price pair $(w^{ci}_\alpha, w^{ci}_\beta)$ are also mutual best replies on the whole domain of strategies, i.e, given price $w^{ci}_\alpha$, platform $\beta$ does not have an incentive to change to price $\overline{w}_\beta$ which will result in another configuration and a higher profit, and vice versa. Formally, we show that $w^{ci}_\beta$ beats any strategy $w_\beta$ in the projection $R_I \cup R_{II} \cup R_{IV}$ against $w^{ci}_\alpha$ and vice versa.

We first fix $w^*_\beta = w^{ci}_\beta$ and show that platform $\alpha$ has no incentive to deviate to any price $\overline{w}_\alpha$. We note that it is not possible for platform $\alpha$ to come up with prices which will result in either configuration $C_I$ or $C_{II}$ because $w^*_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.[10] We therefore check to see if platform $\alpha$ deviates to a covered but preempted market, i.e, configuration $C_{IV}$. We denote the profit of platform $\alpha$ under the price pair $(w^*_\alpha, w^*_\beta)$ as $\pi^*_\alpha$ and that under the pair $(\overline{w}_\alpha, w^*_\beta)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

[8] $\partial^2 d(\overline{\gamma})/\partial^2 \overline{\gamma} = ((33fy_\beta^2 - 27y_\beta^2 - 8y_\alpha f^2 y_\beta + 6fy_\alpha^2 - 54y_\alpha y_\beta - 39y_\alpha f y_\beta + 4y_\alpha^2 f^2 + 4y_\beta^2 f^2)f)/(108(a(y_\alpha - y_\beta)))$.

[9] An interior solution refers to the instance when $w^*_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.

[10] The fact that $w^*_\beta$ is an interior solution implies a covered market will result for any value $\overline{w}_\alpha$.

1. *Platform $\alpha$ has no incentive to deviate to configuration $C_{IV}$.* If platform $\alpha$ chooses to deviate to a configuration where all CPs subscribe to it, the best price it can offer is denoted by $\overline{w}_\alpha$ and is given by,

$$\overline{w}_\alpha = \text{argmax } \pi_\alpha(w_\alpha, w^*_\beta),$$
$$\text{s.t. } w_\alpha \leq q_\alpha(\overline{\gamma} - a)(y_\alpha - y_\beta) + w^*_\beta.$$

The constraint in the above maximization problem reflects the fact that all content providers should prefer platform $\alpha$ to platform $\beta$ for configuration $C_{IV}$ to occur. Since $\pi_\alpha$ is linear and increasing in $w_\alpha$, $\overline{w}_\alpha = (\overline{\gamma} - a)q_\alpha(y_\alpha - y_\beta) + w^*_\beta$. Under this price $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[11] Moreover $d(\overline{\gamma})$ has a single root at $\overline{\gamma} = a\frac{5f+18}{5f+6}$. Thus for all values of $\overline{\gamma}$, the following inequality holds, $d(\overline{\gamma}) \geq 0$. Consequently platform $\alpha$ has no incentive to deviate to configuration $C_{IV}$.

We now fix $w^*_\alpha = w^{ci}_\alpha$ and show that platform $\beta$ has no incentive to deviate to any price $\overline{w}_\beta$ in any other configuration. We denote the profit of platform $\beta$ under the price pair $(w^*_\alpha, w^*_\beta)$ as $\pi^*_\beta$ and that under the pair $(\overline{w}_\beta, w^*_\alpha)$ as $\overline{\pi}_\beta$. We denote the difference $\pi^*_\beta - \overline{\pi}_\beta$ as $d(\overline{\gamma})$.

1. *Platform $\beta$ has no incentive to deviate to configuration $C_I$.* We show that the best response given $w^*_\alpha$, such that configuration $C_I$ emerges, will yield a lower profit. Let $\overline{w}_\beta$ denote the best response under $C_I$ given $w^*_\alpha$. It is given by,

$$\overline{w}_\beta = \text{argmax } \pi_\beta(w^*_\alpha, w_\beta),$$
$$\text{s.t. } w_\beta \geq \frac{y_\beta w^*_\alpha(q_\alpha + q_\beta)}{(q_\alpha y_\alpha + y_\beta q_\beta)}.$$

For this configuration to occur the lowest quality content provider should not join platform $\alpha$, which implies $w^*_\alpha > (\overline{\gamma} - a)(q_\alpha y_\alpha + q_\beta y_\beta)$. This implies that the configuration is possible only if $\frac{\overline{\gamma}}{a} < \frac{7f(y_\alpha - y_\beta) + 36y_\alpha - 9y_\beta}{7f(y_\alpha - y_\beta) + 12y_\alpha + 15y_\beta}$. We denote this bound by $\tilde{\gamma}$. Moreover, from section A.2 we know that $w^*_\alpha$ is defined only if $\frac{\overline{\gamma}}{a} > \frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta}$. We denote this upper bound by $\hat{\gamma}$. Therefore, configuration $C_{IV}$ can occur only if $\hat{\gamma} < \overline{\gamma} < \tilde{\gamma}$. The function $d(\overline{\gamma})$ is a convex function in $\overline{\gamma}$, because, $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[12] Moreover $d(\overline{\gamma})$ has two roots at $\gamma_1$ and $\gamma_2$. These are given explicitly below,

$$\gamma_1 = \frac{a(Q(f, y_\beta) + \sqrt{(8f^2 + 216 + 96f)y_\alpha + 36(18fy_\beta^2 + 6f^2y_\beta^2 - 6y_\alpha f^2 y_b + 36y_\alpha f y_\beta)})}{((36 - 30f + 67f^2)y_\beta + (8f^2 + 72 + 48f)y_\alpha)}, \quad (33)$$

$$\gamma_2 = \frac{a(Q(f, y_\beta) + \sqrt{(8f^2 + 216 + 96f)y_\alpha - 36(18fy_\beta^2 + 6f^2y_\beta^2 - 6y_\alpha f^2 y_\beta + 36y_\alpha f y_\beta)})}{((36 - 30f + 67f^2)y_b + (8f^2 + 72 + 48f)y_\alpha)}. \quad (34)$$

where $Q(f, y_\beta) = (67f^2 + 102f + 108)y_\beta$. Thus for $\overline{\gamma} \leq \gamma_2$, we have $d(\overline{\gamma}) \geq 0$. It is also the case that $\gamma_2 \geq \tilde{\gamma} \geq \hat{\gamma}$ when $\frac{y_\alpha}{y_\beta} \leq \frac{f+9}{f}$. Therefore for $\frac{y_\alpha}{y_\beta} \leq \frac{f+9}{f}$ platform $\beta$ has no incentive to deviate. For $\frac{y_\alpha}{y_\beta} > \frac{f+9}{f}$ , $\tilde{\gamma} < \hat{\gamma}$ which implies that configuration $C_{IV}$ is not possible. Thus given $w^*_\alpha$, platform $\beta$ has no incentive to deviate to a price that results in $C_{IV}$.

[11] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{1}{486a}(6 + 5f)^2(y_\alpha - y_\beta)f$.

[12] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{(y_\alpha - y_\beta)f((36 - 30f + 67f^2)y_\beta + (8f^2 + 72 + 48f)y_\alpha)}{486(y_\beta + 2y_\alpha)a}$.

2. *Platform $\beta$ has no incentive to deviate to configuration $C_{II}$.* For this configuration to occur the lowest quality content provider should not join platform $\beta$, which implies $w_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. Therefore, platform $\beta's$ best price under this configuration is formally given by,

$$\overline{w}_\beta = \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),$$
$$\text{s.t. } w_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta.$$

The profit function $\pi_\beta$ is concave in $w_\beta$. An interior solution to the above maximization problem exists only if $\overline{w}_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. One can show that this happens only if $\overline{\gamma} < \check{\gamma}$ where $\check{\gamma} = a\frac{(20f(y_a - y_b) + 9y_\alpha + 18y_\beta)}{(20f(y_\alpha - y_\beta) - 3y_\alpha + 30y_\beta)}$. But configuration $C_{III}$ with an interior solution is only defined for $\hat{\gamma} < \overline{\gamma} < a\frac{5f+18}{5f+6}$. Since $\hat{\gamma} > \check{\gamma}$, a maximum does not exist and the supremum of the profit function under this configuration is that given under the price $\overline{w}_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. The function $d(\overline{\gamma})$, under this price, is a convex function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[13] Moreover, $d(\overline{\gamma})$ has a single root at $\hat{\gamma}$. Therefore $d(\overline{\gamma}) > 0$ for all $\hat{\gamma} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$. This implies that given $w_\alpha^*$, platform $\beta$ has no incentive to deviate to a price that results in configuration $C_{II}$.

3. *Platform $\beta$ has no incentive to deviate to configuration $C_{IV}$ where all CPs migrate to platform $\alpha$.* For this configuration to occur the lowest quality content provider should not join platform $\beta$ but platform $\alpha$. This implies $w_\beta \geq (\overline{\gamma} - a)(y_\beta - y_\alpha) + w_\alpha^*$. Therefore, platform $\beta's$ best price under this configuration is formally given by,

$$\overline{w}_\beta = \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),$$
$$\text{s.t. } w_\beta \geq (\overline{\gamma} - a)(y_\beta - y_\alpha) + w_\alpha^*.$$

For this configuration to occur $\overline{\gamma} \geq \tilde{\gamma}$.[14] The function $d(\overline{\gamma})$ is a convex function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$.[15] Moreover, $d(\overline{\gamma})$ has a single root at $a\frac{5f+18}{5f+6)}$. Therefore $d(\overline{\gamma}) \geq 0$ for all $\overline{\gamma}$, and in particular when $\overline{\gamma} \geq \tilde{\gamma}$. This implies that given $w_\alpha^*$, platform $\beta$ has no incentive to deviate to a price that results in configuration $C_{IV}$.

4. *Platform $\beta$ has no incentive to deviate to configuration $C_{IV}$ where all CPs migrate to platform $\beta$.* For this configuration to occur the highest quality content provider should join platform $\beta$ . This implies $w_\beta \leq (\overline{\gamma} + a)(y_\beta - y_\alpha) + w_\alpha^*$. Therefore, platform $\beta's$ best price under this configuration is formally given by,

$$\overline{w}_\beta = \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),$$
$$\text{s.t. } w_\beta \leq (\overline{\gamma} - a)(y_\beta - y_\alpha) + w_\alpha^*.$$

The function $d(\overline{\gamma})$ is a convex function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} > 0$. Moreover, $d(\overline{\gamma})$ has a single root at $a\frac{5f-18}{5f+6)}$. Therefore $d(\overline{\gamma}) \geq 0$ for all $\overline{\gamma}$, and in particular when this configuration occurs. This implies that given $w_\alpha^*$, platform $\beta$ has no incentive to deviate to a price that results in configuration $C_{IV}$.

□

We finally show that configuration $C_{IV}$ exists. We give the necessary conditions for its existence together with the possible price characterizations in this configuration.

13 $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{(((f+3)((f+3/2)^2-63/4))^2 y_\beta^2 + ((f+3)((f+3/2)^2-63/4))y_\alpha y_\beta + 4f^2(f+3)^2 y_\alpha^2)}{486fa(y_\alpha - y_\beta)}$.

14 If $\overline{\gamma} < \tilde{\gamma}$ then we cannot have a covered market where all CP's patronize platform $\alpha$.

15 $\partial^2 d(\overline{\gamma})/\partial^2\overline{\gamma} = (25f^2 + 60f + 36)(y_\alpha - y_\beta)f/486a$.

LEMMA 6. *Given a tuple* $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$*, there exists an equilibrium price pair* $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{IV}$ *such that*

1. $w_\beta^* > (\overline{\gamma} - a)y_\beta$, $w_\alpha^* = (\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha)$ *only if,*
$$\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta} \text{ and } y_\alpha \geq \frac{f+9}{f} y_\beta.$$
2. $w_\beta^* = (\overline{\gamma} - a)y_\beta$, $w_\alpha^* = (\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha)$ *only if,*
$$\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta} \text{ and } y_\alpha \geq \frac{f+9}{f} y_\beta.$$
3. $w_\beta^* = (\overline{\gamma} - a)y_\beta - c(\overline{\gamma} - a)y_\beta$, $w_\alpha^* = \frac{1}{3}(\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha) - c(\overline{\gamma} - a)y_\beta$ *only if,*
$$\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta - 9y_\beta c}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta - 9y_\beta c} \text{ and } y_\alpha \geq 10y_\beta - 9cy_\beta,$$
$$\text{where } 0 < c < 1 \text{ and } y_\alpha \geq \frac{f+9-9c}{f} y_\beta.$$
4. $w_\beta^* = 0$, $w_\alpha^* = \frac{2}{3}(\overline{\gamma} - a)(y_\alpha - y_\beta)$ *only if,*
$$\frac{9+2f}{3+2f} \leq \frac{\overline{\gamma}}{a} < \infty.$$

*Proof.* From section A.2, the condition in (30) characterizes the equilibrium price pairs that exist if configuration $C_{IV}$ is exogenously assumed. We show a subset of this characterization is a subgame Nash equilibrium for the range of values of $\overline{\gamma}$ stated in the Lemma.

*Proving case 1 :* $w_\beta^* > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$, $w_\alpha^* = (\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha)$.

Let $(w_\alpha^*, w_\beta^*)$ be a price pair that satisfies the condition in 30 where $w_\beta^* > (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$. We fix $w_\alpha^*$ and check whether platform $\beta$ has an incentive to deviate to configuration $C_{III}$. Note that this is the only configuration that platform $\beta$ can deviate too; since $w_\alpha^* = (\overline{\gamma} - a)(q_\alpha y_\alpha + q_\beta y_\beta)$ the lowest quality content provider will join at least one platform. Thus platform $\beta$ can only deviate to a covered market configuration.

We denote the profit of platform $\beta$ under the price pair $(w_\alpha^*, w_\beta^*)$ as $\pi_\beta^*$ and that under the pair $(\overline{w}_\beta, w_\alpha^*)$ as $\overline{\pi}_\beta$. We denote the profit difference $\pi_\beta^* - \overline{\pi}_\beta$ as $d(\overline{\gamma})$. Platform $\beta$ maximizes its profit function to find the best price $\overline{w}_\beta$ that will yield configuration $C_{III}$ given the tuple $(y_\alpha, y_\beta, \overline{\gamma}, a)$. The maximization problem has a constraint which ensures that the price is less than the value gained by the lowest quality CP.

$$\text{Let } \overline{w}_\beta = \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),$$
$$\text{s.t. } 0 \leq w_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta.$$

It follows that $0 \leq \overline{w}_\beta < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$ only if $\overline{\gamma} > a$ and $\frac{y_\alpha}{y_\beta} < \frac{f+9}{f}$. Consequently, market configuration $C_{III}$ is possible with this price only if $\frac{y_\alpha}{y_\beta} < \frac{f+9}{f}$ since we assume in the problem formulation that $\overline{\gamma} > a$. Under the price $\overline{w}_\beta$ the function $d(\overline{\gamma})$ is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[16] Moreover, $d(\overline{\gamma})$ has a single root $a$. Therefore for all $\overline{\gamma} > a$ and $y_\alpha < y_\beta \frac{f+9}{f}$ platform $\beta$ will deviate to configuration $C_{III}$. This suggests that we potentially could have a preempted solution when $y_\alpha \geq y_\beta \frac{f+9}{f}$.

We fix $w_\beta^*$ and check whether platform $\alpha$ has an incentive to deviate to configuration $C_I$ or $C_{II}$ when $\frac{f+9}{f}$. We only consider those two configurations because configuration $C_{III}$ is not possible given $w_\beta^* > (\overline{\gamma} - 1)y_\beta$.

[16] $\partial^2 d(\overline{\gamma})/\partial^2\overline{\gamma} = (-y_\alpha f + (9+f)y_\beta)^2 f/(216a(y_\alpha - y_\beta))$.

We denote the profit of platform $\alpha$ under the price pair $(w^*_\alpha, w^*_\beta)$ as $\pi^*_\alpha$ and that under the pair $(\overline{w}_\alpha, w^*_\beta)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

$$\text{Let } \overline{w}_\alpha = \text{argmax } \pi_\alpha(w^*_\beta, w_\alpha),$$
$$\text{s.t. } w_\alpha \leq w^*_\beta\left(\frac{q_\beta y_\beta + q_\alpha y_\alpha}{(q_\alpha + q_\beta) y_\beta}\right).$$

Let $\overline{w}^{ur}_\alpha$ be the interior solution. It follows that this solution exists whenever $w^*_\beta \geq \frac{(q_\alpha+q_\beta)y_\beta \overline{w}^{ur}_\alpha}{y_\beta q_\beta + q_\alpha y_\alpha}$. We denote this bound by $w^{**}_\beta$. Therefore an interior solution $\overline{w}^{ur}_\alpha$ results in market configuration $C_I$ only if $w^*_\beta \geq w^{**}_\beta > (\overline{\gamma} - a)(q_\alpha + q_\beta) y_\beta$. One can show that this occurs only if $\overline{\gamma} < \frac{a(4f(y_\alpha - y_\beta)+18y_\alpha+9y_\beta)}{(4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta)}$. We denote this bound as $\overline{\gamma}^*$. The function $d(\overline{\gamma})$ under this price is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$. [17] Moreover $d(\overline{\gamma})$ has a single root at $\overline{\gamma}^*$. Therefore, for all $\overline{\gamma} < \overline{\gamma}^*$ platform $\beta$ will deviate to configuration $C_I$.

We now investigate the case when $w^*_\beta \in ((\overline{\gamma} - a)(q_\alpha + q_\beta) y_\beta, w^{**}_\beta)$. In this case $\overline{w}_\alpha = \frac{w^*_\beta(y_\beta q_\beta + q_\alpha y_\alpha)}{y_\beta(q_\beta + q_\alpha)}$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{w}^*_\beta)$.. This difference is convex in $w^*_\beta$ since $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{q_\alpha y_\alpha + q_\beta y_\beta}{a y_\beta} > 0$.. This difference has two roots at $r_1$ and $r_2$.[18] Whenever $w^*_\beta \in (r1, r2)$ platform $\alpha$ has an incentive to deviate. It follows that this occurs whenever $r_2 > r1$ which in turn results whenever $\overline{\gamma} < \overline{\gamma}^*$. Therefore if $w^*_\beta \in ((\overline{\gamma} - a)(q_\alpha + q_\beta) y_\beta, w^{**}_\beta)$ , which occurs only if $\overline{\gamma} < \overline{\gamma}^*$ platform $\alpha$ has an incentive to deviate. This implies that there is a potential for a preempted market if $\overline{\gamma} \geq \overline{\gamma}^*$.

We now check whether there is an incentive for platform to deviate to configuration $C_{II}$ or $C_{III}$ where $r_\alpha \in (0,1)$. This can occur whenever $w^*_\beta \in [(\overline{\gamma} - a)(q_\alpha + q_\beta) y_\beta, 2/9(-\overline{\gamma}(2f+3) + a(2f-3))(y_\alpha - y_\beta) f]$. Moreover, this interval is non-empty whenever $\frac{\overline{\gamma}}{a} < \frac{(4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta)}{(4f(y_\alpha-y_\beta)+6y_\alpha+3y_\beta)}$. This implies that platform $\alpha$ would have an incentive to deviate to whenever the former applies since $d(\overline{w}^*_\beta) > 0$ is positive in this range.

Putting all the above results together implies that configuration $C_{IV}$ with the price pair given above is possible only if $\overline{\gamma} > \overline{\gamma}^*$.

*Proving case 2:* $w^*_\beta = (\overline{\gamma} - a) y_\beta$, $w^*_\alpha = \frac{1}{3}(\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha)$

The first part of the proof where we fix $w^*_\alpha$ and check whether platform $\beta$ has an incentive to deviate to configuration $C_{III}$ is exactly the same as in the previous case. We fix $w^*_\beta$ and check whether platform $\alpha$ has an incentive to deviate to configuration $C_{III}$ when $\frac{y_\alpha}{y_\beta} \geq \frac{f+9}{f}$. We only consider these configuration because configuration $C_I$ and $C_{II}$ are not possible given $w^*_\beta = (\overline{\gamma} - a)(q_\alpha + q_\beta) y_\beta$.

We denote the profit of platform $\alpha$ under the price pair $(w^*_\alpha, w^*_\beta)$ as $\pi^*_\beta$ and that under the pair $(\overline{w}_\alpha, w^*_\beta)$ as $\overline{\pi}_\alpha$. We denote the difference $\pi^*_\alpha - \overline{\pi}_\alpha$ as $d(\overline{\gamma})$.

$$\text{Let } \overline{w}_\alpha = \text{argmax } \pi_\alpha(w^*_\beta, w_\alpha),$$
$$\text{s.t. } w_\alpha > (\overline{\gamma} - a)(q_\beta y_\beta + q_\alpha y_\alpha).$$

It follows that $\overline{w}_\alpha$ exists whenever $\overline{\gamma} < \overline{\gamma}^*$. It is also the case that function $d(\overline{\gamma})$ is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[19] Moreover, $d(\overline{\gamma})$ has a single root $\overline{\gamma}^*$. Therefore for all $\overline{\gamma} \neq \overline{\gamma}^*$ platform $\beta$ will deviate

[17] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{-(36y_\alpha^2+16y_\alpha^2 f^2+48y_\alpha^2 f-24y_\alpha f y_\beta-32y_\alpha f^2 y_\beta+36y_\beta y_\alpha+9y_\beta^2+16f^2 y_\beta^2-24f y_\beta^2)}{108fa(2y_\alpha+y_\beta)}$.

[18] $r_1 = (\overline{\gamma} - a)(q_\alpha + q_\beta) y_\beta$, and $r_2 = \frac{2}{3}\frac{f y_\beta((2f(\overline{\gamma}-a)+3a)y_\beta+(2f(a-\overline{\gamma}+6a)y_\alpha))}{2y_\alpha+y_\beta}$.

[19] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{-1(y_\alpha f-(9+f)y_\beta)^2 f}{216a(y_\alpha-y_\beta)}$.

to configuration $C_{III}$. In particular, when $\overline{\gamma} < \overline{\gamma}^*$ platform $\alpha$ will always deviate since configuration $C_{III}$ is defined for that range. This implies that a preempted market with prices $(w_\alpha^*, w_\beta^*)$ occurs only if $\overline{\gamma}^* \leq \overline{\gamma}$ and $\frac{y_\alpha}{y_\beta} \geq \frac{f+9}{f}$.

*Proving case 3:* $w_\alpha^* = (\overline{\gamma}-a)(q_\alpha y_\alpha + q_\beta y_\beta) - c(\overline{\gamma}-a)y_\beta$ *and* $w_\beta^* = (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta - c(\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta$, *where* $c \in [0,1)$.

We fix $w_\alpha^*$ and check whether platform $\beta$ has an incentive to deviate to configuration $C_{III}$. Note that this is the only configuration that platform $\beta$ can deviate too since $w_\alpha^* < (\overline{\gamma}-a)(q_\alpha y_\alpha + q_\beta y_\beta)$ which implies that the lowest quality content provider will join at least one platform.

We denote the profit of platform $\beta$ under the price pair $(w_\alpha^*, w_\beta^*)$ as $\pi_\beta^*$ and that under the pair $(\overline{w}_\beta, w_\alpha^*)$ as $\overline{\pi}_\beta$. We denote the difference, $\pi_\beta^* - \overline{\pi}_\beta$ as $d(\overline{\gamma})$. Platform $\beta$ maximizes the following profit function to find the best price $\overline{w}_\beta$ that will yield configuration $C_{III}$ given the tuple $(y_\alpha, y_\beta, \overline{\gamma})$.

$$\text{Let } \overline{w}_\beta = \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),$$
$$\text{s.t. } 0 < w_\beta < (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta(1-c).$$

It follows that $\overline{w}_\beta \in (0, (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta(1-c))$ whenever $\frac{y_\alpha}{y_\beta} \leq \frac{(9-9c+f)}{f}$. It is also the case that function $d(\overline{\gamma})$ is a concave function of $\overline{\gamma}$, because $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} < 0$.[20] Moreover, $d(\overline{\gamma})$ has a single root $a$. Therefore, for all $\overline{\gamma} > a$ and $\frac{y_\alpha}{y_\beta} < \frac{(9-9c+f)}{f}$ platform $\beta$ will deviate to configuration $C_{III}$. This suggests that we potentially could have a preempted solution when $\overline{\gamma} > a$ and $\frac{y_\alpha}{y_\beta} > \frac{(9-9c+f)}{f}$ .

We now fix $w_\beta^* = (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta - c(\overline{\gamma}-a)y_\beta$ where $c \in [0,1)$ and check whether platform $\alpha$ has an incentive to deviate to configuration $C_{III}$ . We again consider only this configuration because configuration $C_I$ and $C_{II}$ are not possible given $w_\beta^* < (\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta$.

$$\text{Let } \overline{w}_\alpha = \text{argmax } \pi_\alpha(w_\beta^*, w_\alpha),$$
$$\text{s.t. } w_\alpha > (\overline{\gamma}-a)\frac{q_\beta y_\beta + q_\alpha y_\alpha}{3} - c(\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta.$$

It follows that $\overline{w}_\alpha > (\overline{\gamma}-a)(q_\beta y_\beta + q_\alpha y_\alpha) - c(\overline{\gamma}-a)(q_\alpha+q_\beta)y_\beta$ and results in configuration $C_{III}$ only if $\frac{\overline{\gamma}}{a} \geq \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta c-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha-9y_\beta c+3y_\beta}$. Therefore platform $\alpha$ deviates only in the above case. Putting all the above results together we find that this price configuration holds whenever, $\frac{\overline{\gamma}}{a} < \frac{4f(y_\alpha-y_\beta)+18y_\alpha-9y_\beta c-9y_\beta}{4f(y_\alpha-y_\beta)+6y_\alpha-9y_\beta c+3y_\beta}$ and $\frac{y_\alpha}{y_\beta} \geq \frac{9+f-9c}{f}$.

*Proving case 4:* $w_\beta^* = 0$, $w_\alpha^* = q_\alpha(\overline{\gamma}-a)(y_\alpha-y_\beta)$ .

We proceed in a similar way to that used in proving case 3. We fix $w_\alpha^*$ and check whether platform $\beta$ has an incentive to deviate to configuration $C_{III}$. Note that this is the only configuration that platform $\beta$ can deviate too since $w_\alpha^* < (\overline{\gamma}-a)(q_\alpha y_\alpha + q_\beta y_\beta)$. This means that platform $\beta$ maximizes its profit function to find the best price $\overline{w}_\beta$ that will yield configuration $C_{III}$ given the tuple $(y_\alpha, y_\beta, \overline{\gamma}, a)$.

$$\text{Let } \overline{w}_\beta = \text{argmax } \pi_\beta(w_\alpha^*, w_\beta),$$
$$\text{s.t. } w_\beta < 0.$$

[20] $\frac{\partial^2 d(\overline{\gamma})}{\partial^2(\overline{\gamma})} = \frac{-1}{216a}\frac{f(f(y_\alpha-y_\beta)+9y_\beta(c-1))^2}{a(y_\alpha-y_\beta)}$.

However any price $w_\beta < 0$ is dominated by $w_\beta = 0$ thus platform $\beta$ has no incentive to deviate for any $\overline{\gamma}$ .

We fix $w_\beta^* = 0$ and check whether platform $\alpha$ has an incentive to deviate to configuration $C_{III}$. We again consider these configuration because configuration $C_I$ and $C_{II}$ are not possible given $w_\beta^* < (\overline{\gamma} - a)(q_\alpha + q_\beta)y_\beta$.

$$\text{Let } \overline{w}_\alpha = \text{argmax } \pi_\alpha(w_\beta^*, w_\alpha), \quad \text{s.t. } w_\alpha > q_\alpha(\overline{\gamma} - a)(y_\alpha - y_\beta).$$

It follows that $\overline{w}_\alpha > q_\alpha(\overline{\gamma} - a)(y_\alpha - y_\beta)$ only if $\frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$. This implies that when $\frac{\overline{\gamma}}{a} \geq \frac{2f+9}{2f+3}$ this price structure and market configuration are possible.

□

In this Appendix we have shown that there exists equilibrium price pairs that are Nash equilibrium in the price subgame. Moreover, we have shown the market configurations in which they occur and the conditions for them to occur. In particular, we have shown that each of the configurations $C_{II}$, $C_{III}$, and $C_{IV}$ exists.

**A.3.1. CP Pricing Equilibrium for $y_\alpha > y_\beta > 0$** In this section, we provide results showing that given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta > 0$, there exists a pure strategy price SPE pair $(w_\alpha^*, w_\beta^*)$. In addition, we prove that just one market configuration is feasible in the price subgame Nash equilibrium, and for market configurations $C_{II}$ and $C_{III}$, the price characterizations are unique. Specifically, we show the conditions under which particular market configurations arise as a function of the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$.

Our results show that the uncovered market configuration $C_I$ does not occur at an SPE. In this configuration no CPs join the low-quality platform even though it has positive quality. We show that there exists a profitable price deviation by the low-quality platform that involves CPs joining this platform. On the other hand, we show that given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ one of the other configurations, $C_{II}$, $C_{III}$ or $C_{IV}$, will emerge. In doing so, we determine the set of parametric values $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ for which these different configurations exist.

We prove the existence of the price SPE constructively. To do that, we first identify candidate equilibrium price pairs in each possible market configuration (see Appendix A.2), and then check whether these price equilibrium pairs are indeed Nash equilibria of the price subgame (see Appendix A.3). We do so by verifying that the equilibrium price candidates are best replies on the whole domain of strategies: That is, not only are they best responses in their respective market configurations but also best replies if the other market configurations are taken into account.

We now present a theorem showing that for any tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ a price subgame Nash equilibrium exists, and only one market configuration is feasible. In addition, for market configurations $C_{II}$ and $C_{III}$, the price characterizations are unique.

THEOREM 6. *Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ that satisfies that $y_\alpha > y_\beta > 0$, there exists a Nash equilibrium pair $(w_\alpha^*, w_\beta^*)$ in the price subgame. Moreover, letting $\tau_1 = \frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}$, $\tau_2 = \frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta}$, $\tau_3 = \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}$, and $\tau_4 = \frac{5f+18}{5f+6}$, the resulting market configuration is unique and the following holds:*

1. *If $1 < \frac{\overline{\gamma}}{a} < \tau_1$, then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{II}$.*
2. *If $\tau_1 \leq \frac{\overline{\gamma}}{a} < \min\{\tau_2, \tau_3\}$ then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{III}$.*
3. *If $\tau_2 < \frac{\overline{\gamma}}{a} < \tau_4$ then the equilibrium price pair is unique and $(w_\alpha^*, w_\beta^*) \in \mathcal{R}_{III}$.*

4. *If* $\max\{\tau_3, \tau_4\} \leq \frac{\overline{\gamma}}{a}$ *then* $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{IV}$.

*Proof.* Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$, we know from Lemma 2 through 6 that an equilibrium pair $(w^*_\alpha, w^*_\beta)$ exists. Moreover, cases 1, 2, and 3 directly follow from Lemma 3 through 5. In particular,

1. If $1 < \frac{\overline{\gamma}}{a} < \frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}$, then the equilibrium price pair is unique and $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{II}$. This follows from Lemma 3.
2. If $\frac{2f(y_\alpha - y_\beta) + 30y_\alpha - 3y_\beta}{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta} \leq \frac{\overline{\gamma}}{a} < \min\left\{\frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta}, \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}\right\}$ then the equilibrium price pair is unique and $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{III}$. This follows from Lemma 4.
3. If $\frac{2f(y_\alpha - y_\beta) + 18y_\alpha + 9y_\beta}{2f(y_\alpha - y_\beta) + 6y_\alpha + 21y_\beta} < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$ then the equilibrium price pair is unique and $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{III}$. This follows from Lemma 5.
4. If $\max\left\{\frac{5f+18}{5f+16}, \frac{4f(y_\alpha - y_\beta) + 18y_\alpha - 9y_\beta}{4f(y_\alpha - y_\beta) + 6y_\alpha + 3y_\beta}\right\} \leq \frac{\overline{\gamma}}{a} < \infty$ then $(w^*_\alpha, w^*_\beta) \in \mathcal{R}_{IV}$. This follows from Lemma 6.

□

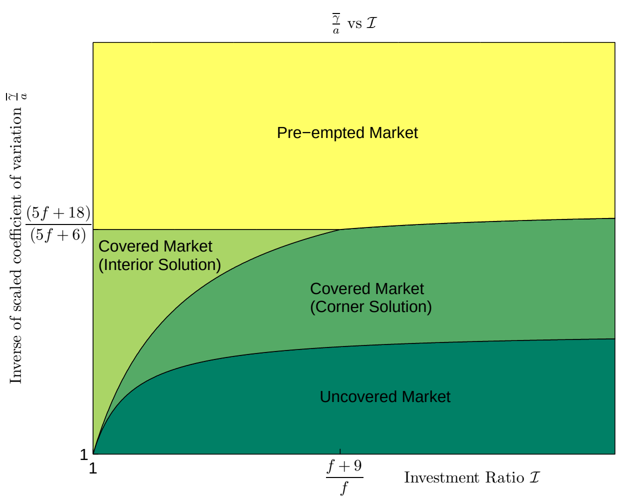


**Figure 12** **Resulting market configurations for the inverse scaled coefficient of variation $\overline{\gamma}/a$ versus platform quality investment ratio $\mathcal{I}$.**

Figure 12 shows the resulting market configurations for different values of the quality investment ratio $\mathcal{I} = y_\alpha / y_\beta$, the inverse of a scaled coefficient of variation, $\overline{\gamma}/a$ and a fixed mass $f$ of consumers. For a fixed $\mathcal{I}$, as $\overline{\gamma}/a$ increases the covered market is more likely. At the extreme, when $\overline{\gamma}/a$ is high, CPs tend to be close to each other with respect to quality. Hence, a decision made by a CP will be mirrored by other CPs and a covered market is likely. On the other hand, for a fixed and low value of $\overline{\gamma}/a$, as the investment ratio increases the two platforms become more differentiated and price competition becomes less intense. This softening of price competition results in an uncovered market because less CPs can afford to join the platforms. However, for a fixed and high value of $\overline{\gamma}/a$, the low heterogeneity in content quality dominates the differentiation effects of the platforms and a preempted covered market is realized as all CPs flock to one platform.

**A.3.2. Proof of Theorem 1** We first show that without loss of generality we can assume platform $\beta$ chooses a price $w_\beta \geq 0$. This will enable us to show that no content provider joins platform $\beta$ and consequently enable us to rule out the existence of configuration $C_{II}$ and $C_{III}$.

LEMMA 7. *Platform $\beta$ charges $w_\beta \geq 0$.*

*Proof.* We first show that $w_\beta \geq 0$ dominates any price $w_\beta < 0$. Assume that $w_\beta < 0$ and $r_\beta > 0$, then platform $\beta$ makes negative revenue on the content provider side. By raising its price to $w_\beta = 0$ it increases its total revenue. This is because the revenue from the content provider side becomes non-negative and the profits on the consumer side increase: This happens across all configurations because $r_\alpha$ is non-decreasing in $w_\beta$. □

Since by Lemma 7, $w_\beta \geq 0$, it follows that any content provider $j$ joining platform $\beta$ will get utility $v_j \leq 0$. Therefore, no content provider has incentive to join platform $\beta$. This implies that market configurations $C_{II}$ and $C_{III}$ where content providers patronize both platforms do not exist. We now show that there trivially exists pure strategy subgame equilibrium price pairs when one platform has zero investment. We show that these prices result in configurations $C_I$ and $C_{IV}$ and give the conditions on $\overline{\gamma}$ for these to occur. We now proceed to prove Theorem 1. *Proof.* We first derive the demand function $r_\alpha(w_\alpha)$. Given $y_\beta = 0$, $y_\alpha > 0$ and Lemma 7, the content provider decisions are as if only one platform is on offer. Therefore, the demand addressed to platform $\alpha$ is equal to the mass of content providers with content quality $\gamma_j$ such that $\gamma_j y_\alpha q_\alpha \geq w_\alpha$ and is given by

$$r_\alpha(w_\alpha) = \frac{1}{2a}\left(\overline{\gamma} + a - \frac{w_\alpha}{q_\alpha y_\alpha}\right).$$

The value $\overline{w}_\alpha$ that maximizes platform alpha's profit problem for platform $\alpha$ is represented as,

$$\begin{aligned}\overline{w}_\alpha = &\operatorname{argmax}\ \pi_\alpha(w_\alpha),\\ &\text{s.t. } w_\alpha \geq (y_\alpha q_\alpha)(\overline{\gamma} - a).\end{aligned}$$

The profit function does not depend on $w_\beta$ so platform $\alpha$ maximizes the above function with respect to $w_\alpha$ and ensuring that $w_\alpha \geq y_\alpha q_\alpha(\overline{\gamma} - a)$. This last constraint reflects the fact that when the constraint binds all content providers are on board; a price lower than this yields no more content providers and results in a loss of revenue. The interior solution for the above maximization is $w_\alpha^* = \frac{1}{9} f y_\alpha(2f(a - \overline{\gamma}) + 3(\overline{\gamma} + a))$, and occurs whenever $1 < \overline{\gamma} < (9 + 2f)/(3 + 2f)$. In this case since $w_\alpha^* > q_\alpha y_\alpha(\overline{\gamma} - a)$ the resulting configuration is $C_I$. The constraint binds when $\overline{\gamma} \geq (9 + 2f)/(3 + 2f)$. In this instance the resulting configuration is $C_{IV}$ since all content providers join platform $\alpha$. □

## A.4. Best reply in the domain $[0, y_h)$

In order to avoid confusion when platform $\beta$ is the high quality firm we will change notation as follows; we label the high(low) quality platform as $h(l)$ and the quality associated with it as $y_{h(l)}$. Given $y_h$, we will compute firm $l$'s best reply. We will show that the profit for the low quality firm is decreasing in $y_l$ across all configurations which are possible given $(\overline{\gamma}, y_h, a)$. This will help us infer that the low quality platform chooses 0 as its best response. Since the choice of $y_l$ by the low quality firm determines the market configuration we define the critical limits for which the various configurations exist given $y_h$.

- Market is uncovered, with positive masses of consumers on both platforms, in the in the price subgame whenever,

$$y_l < 2y_h \frac{a(f+15)-(f+9\overline{\gamma})}{(9-2f)\overline{\gamma}+a(2f+3)}. \tag{35}$$

- Market is covered and a corner solution applies in the price subgame whenever,

$$y_l \in \left[2y_h \frac{a(f+15)-(f+9\overline{\gamma})}{(9-2f)\overline{\gamma}+a(2f+3)}, 2y_h \frac{a(f+9)-(f+3)\overline{\gamma}}{(21-2f)\overline{\gamma}-(9-2f)a}\right], \tag{36}$$
$$\text{if } 1 < \tfrac{\overline{\gamma}}{a} \leq \tfrac{f+15}{f+9},$$

$$y_l \in \left[0, 2y_h \frac{(f+9)a-(f+3)\overline{\gamma}}{(21-2f)\overline{\gamma}-(9-2f)a}\right], \tag{37}$$
$$\text{if } \tfrac{f+15}{f+9} < \tfrac{\overline{\gamma}}{a} \leq \tfrac{5f+18}{5f+6}.$$

$$y_l \in \left[0, 2y_h \frac{(2f+9)a-(2f+3)\overline{\gamma}}{(3-4f)\overline{\gamma}+(9+4f)a}\right], \tag{38}$$
$$\text{if } \tfrac{5f+18}{5f+6} < \tfrac{\overline{\gamma}}{a} \leq \tfrac{2f+9}{2f+3}.$$

- Market is covered and an interior solution applies in the price subgame whenever,

$$y_l \in \left(2y_h \frac{(-(f+3)\overline{\gamma}+(f+9)a)}{(21-2f)\overline{\gamma}-(9-2f)a}, y_h\right), \text{ if } 1 < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}. \tag{39}$$

- Market is preempted whenever,

$$y_l \in \left[2y_h \frac{(2f+9)a-(2f+3)\overline{\gamma}}{(3-4f)\overline{\gamma}+(9+4f)a}, y_h\right), \text{ if } \tfrac{5f+18}{5f+6} \leq \tfrac{\overline{\gamma}}{a} < \tfrac{9+2f}{3+2f}. \tag{40}$$

$$\frac{\overline{\gamma}}{a} \geq \frac{9+2f}{3+2f}. \tag{41}$$

We now show that given the tuple $(\overline{\gamma}, y_h, a)$ the profit function $\pi_l$ is decreasing in every configuration that it is defined.

Lemma 8. *Given $(y_h, \overline{\gamma}, a)$, $f \geq 3/5$ and $y_l \in [0, y_h)$ , the profit function $\pi_l(y_l, y_h)$ is decreasing in $y_l$ for all market configurations for which it is defined.*

*Proof.* We show that for each configuration the revenue function $r_l = \pi_l(y_l, y_h) + c(y_l)$ is decreasing in $y_l$.

*Uncovered Configuration $C_I$:* Let the revenue function in this configuration be defined by $r_{ui}$, one can by show that for $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$ and $y_l$ in the set defined in (35), $\frac{\partial r_{ui}}{\partial y_l} < 0$. Hence the revenue function in this configuration is decreasing in $y_l$.

*Covered Configuration with interior solution $C_{III}$:* Let the revenue function in this configuration be defined by $r_{ci}$. One can also show that for $1 < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$ and $y_l$ in the set defined in 39, the derivative of the above function, $\frac{\partial r_{ci}}{\partial y_l} < 0$. Therefore the profit function is decreasing in $y_l$ when $y_l$ lies in the set specified in (39).

*Covered Configuration with corner solution $C_{III}$:* Let the revenue function in this configuration be defined by $r_{cc}$. One can show that for $1 < \frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$ and $y_l$ in the sets defined in (36), (37) and (38) the derivative of the above function, $\frac{\partial r_{cc}}{\partial y_l} < 0$.

*Pre-empted Configuration $C_{IV}$:* Let the revenue function in this configuration be defined by $r_p$. The derivative of the above function, $\frac{\partial r_p}{\partial y_l} = -2/9f^2\overline{\gamma}$. The above derivative is negative therefore the profit function is decreasing in $y_l$ when this configuration is defined. □

LEMMA 9. *Given the tuple $(y_\alpha, \overline{\gamma}, a, f)$, $f \geq 3/5$ and the domain $[0, y_\alpha)$, then $B_\beta(y_\alpha) = 0$.*

*Proof.* We show that given $(y_h, \overline{\gamma}, a)$ platform $l$ will prefer not to invest. Specifically, we show that the profit function $\pi_l$ is continuous in $y_l$ over the domain $[0, y_h)$. This coupled with Lemma 8 implies that platform $l$ picks the lowest quality as the Lemma claims. We split the domain in which $\frac{\overline{\gamma}}{a}$ lies into four sections depending on the number and type of market configurations that are possible. We show that in each section the profit function is continuous.

*Case I.* $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$

Four market configurations are possible when $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$; these are uncovered ($C_I$), uncovered ($C_{II}$) with both platforms participating, covered with a corner solution and covered with an interior solution (both of which are in configuration $C_{III}$). Given a $\frac{\overline{\gamma}}{a}$ in the above range, the domain $[0, y_h)$ in which $y_l$ lies can be partitioned into three sets, each of which corresponds to one of the latter three market configurations. These partitions are captured in (35), (36) and (39). By Lemma 8, we know that profits are decreasing in $y_l$ for each partition. We will first show that the value of the profit function in the partition defined in (35), is larger than any profit attained in the partition defined in (36). Similarly, we show that any profit attained when $y_l$ lies in the partition defined by the constraint in (39) is not greater than that attained when $y_l$ lies in the partition specified by (36). Lastly we show that the profit of a platform in configuration $C_{II}$ tends to that in configuration $C_I$ as $y_l \to 0$ and is in fact equal at the limit.

To show the first result we compare the infimum value of the profit function in the uncovered configuration to the highest possible profit attained when platform $l$ chooses $y_l$ such that a covered market with a corner solution results (i.e, $y_l$ is in the set specified in (36)). Let $y_l^{cc} = 2y_h \frac{a(f+15)-(f+9\overline{\gamma})}{(9-2f)\overline{\gamma}+a(2f+3)}$, it follows that $\lim_{y_l \to y_l^{cc}} \pi_l^u(y_l, y_h) = \pi_l^{cc}(y_l^{cc}, y_h)$ (Since $\pi_l^u$ is right continuous, the limit exists). Since $\pi_l^u(y_l, y_h) > \pi_l^u(y_l^{cc}, y_h)$ when $y_l$ satisfies the inequality in (35), it also follows from Lemma 8 that $\pi_l^u(y_l, y_h) > \pi_l^{cc}(\tilde{y}, y_h)$ when $\tilde{y}$ lies in the set specified in (36).

To show the second result, we compare the lowest value of the profit function in the covered configuration with a corner solution to the supremum profit value attained when platform $l$ chooses $y_l$ such that a covered market with an interior solution results. The interval over which the covered configuration with an interior solution, $C_{III}$, is defined is open. Let $y_l^{ci} = 2y_h \frac{-(3\overline{\gamma}+(f+9)a)}{(21-2f)\overline{\gamma}-(9-2f)a}$, we define the supremum of $\pi_l^{ci}(y_l, y_h)$ over the range in which this configuration is defined as $\pi_l^{ci}(y_l^{ci}, y_h)$. We note that $y_l^{ci}$ is the infimum of the interval over which this configuration is defined, therefore $\lim_{y_l \to y_l^{ci}} \pi_l^{ci}(y_l, y_h) = \pi_l^{ci}(y_l^{ci}, y_h)$ since $\pi_l^{ci}(y_h, y_l)$ is left continuous. By plugging in $y_l = y_l^{ci}$ into $\pi_l^{cc}(y_l, y_h)$ we note that $\pi_l^{cc}(y_l^{ci}, y_h) = \pi_l^{ci}(y_l^{ci}, y_h)$. Therefore, it follows from Lemma 8, that $\pi_l^{cc}(y_h, y_l) > \pi_l^{ci}(\tilde{y}, y_h)$ when $y_l$ satisfies the constraint in (36) and $\tilde{y}$ satisfies the constraint in (39).

Finally, since $\pi_l^u(y_l, y_h)$ is left continuous by plugging $y_l = 0$ to the function $\pi_l^u(y_l, y_h)$ we show that $\lim_{y_l \to 0} \pi_l^u(y_l, y_h) = \pi_l^{ui}(0, y_h)$. Where $\pi_l^{ui}(0, y_h)$ is the profit function when configuration $C_I$ is defined.

*Case II.* $\frac{f+15}{f+9} \leq \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$.

In this instance three market configurations are possible depending on the value of $y_h$ and $y_l$; these are uncovered $C_I$, covered with a corner solution, $C_{III}$, and covered with an interior solution, $C_{III}$. Given a $\frac{\overline{\gamma}}{a}$ in the above range, the domain $[0, y_h)$ in which $y_l$ lies can be partitioned into two sets each of which corresponds to the latter two of the three market configurations. These partitions are captured in (36) and (39). We proceed in a similar manner as we did for the previous case. By Lemma 8 we know that profits are decreasing in $y_l$ for each partition. We claim that any profit attained in the partitions defined in (39) is less than that attained by the minimum profit in the partition defined in (36). The proof is exactly the same as that described in case I. We then show that as $y_l \to 0$ the profit of a platform in configuration $C_{III}$ with a corner solution approaches that of the profit in configuration $C_I$ and in the limit when $y_l = 0$ they are equal. Since $\pi_l^{cc}(y_l, y_h)$ is left continuous by plugging $y_l = 0$ to the function $\pi_l^{cc}(y_l, y_h)$ one can show that $\lim_{y_l \to 0} \pi_l^{cc}(y_l, y_h) = \pi_l^{ui}(0, y_h)$. Where $\pi_l^{ui}(0, y_h)$ is the profit function when configuration $C_I$ is defined.

*Case III.* $\frac{5f+18}{5f+6} \leq \frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$.

In this section we need only show that the profit function is continuous across configuration $C_{III}$ with a corner solution and a pre-empted market configuration $C_{IV}$. Indeed these are the only two configurations possible when $y_l > 0$. (We already showed in the previous case that $\lim_{y_l \to 0} \pi_l^{cc}(y_l, y_h) = \pi_l^{ui}(0, y_h)$). To show this result we compare the infimum value of the profit function in the covered configuration with a corner solution to the profit value attained when platform $l$ chooses a $y_l$ such that a pre-empted market results. Note the interval over which the covered configuration with a corner solution, $C_{III}$, is defined is open on its upper limit. Let $y_l^p = 2y_h \frac{(2f+9)a-(2f+3)\overline{\gamma}}{(3-4f)\overline{\gamma}+(9+4f)a}$, we define the infimum of $\pi_l^{cc}(y, y_l)$ over the range in which this configuration is defined as $\pi_l^{cc}(y_l^p, y_h)$. Since $\pi_l^{cc}(y, y_h)$ is right continuous $\lim_{y_l \to y_l^p} \pi_l^{cc}(y_l, y_h) = \pi_l^{cc}(y_l^p, y_h)$. Note $y_l^p$ is the infimum of the range. By plugging in $y_l = y_l^p$ into the profit functions under a covered market (with a corner solution) and a pre-empted market, we find that $\pi_l^{cc}(y_l^p, y_h) = \pi_l^p(y_l^p, y_h)$. This implies that the profit function is continuous across these two market configurations at this point.

*Case IV.* $\frac{2f+9}{2f+3} \leq \frac{\overline{\gamma}}{a} < \infty$.

When $\overline{\gamma}$ falls in the above range only market configuration $C_{IV}$ is possible when $y_l > 0$. We showed in Lemma 8 that the profit function $\pi_l(y_l, y_h)$ is decreasing in $y_l$ in this configuration. So we only need show that $\lim_{y_l \to 0} \pi_l^p(y_l, y_h) = \pi_l^{uic}(0, y_h)$, where $\pi_l^{uic}(0, y_h)$ is the profit function in $C_I$. Since $\pi_l^p(y, y_h)$ is left continuous $\lim_{y_l \to 0} \pi_l^p(y_l, y_h) = \pi_l^p(0, y_h)$. Via simple algebra, one can show that $\pi_l^p(0, y_h) = \pi_l^{uic}(0, y_h)$ which shows that $\pi_l$ is continuous. □

### A.5. Proof of Theorem 2

We first show that a symmetric equilibrium is not feasible. Let $j, i \in \{\alpha, \beta\}$ and $B_i(y_j)$ be the set of $y_i^* \in [0, \infty]$ such that $y_i^* \in \text{argmax}_{y_i \in [0,\infty]} \pi_i(y_i, y_j)$.

Lemma 10. *If $y_j \in [0, \infty]$ then $y_j \notin B_i(y_j)$.*

*Proof.* We show that given $y_j$, platform $i$ never chooses $y_i = y_j \geq 0$ and therefore a symmetric equilibrium is not possible. A symmetric argument applies for the other platform. Assume $y_j \in B_i(y_j)$ so that $y_i = y_j > 0$, then both platforms would make zero profits because of Bertrand competition on both sides of the market. We now check if platforms would prefer $y_i = y_j = 0$ and show that there exists a profitable deviation for platform $i$. There are two cases to consider, when $\overline{\gamma}/a < (2f+9)/(2f+3)$ and $\overline{\gamma}/a \geq (2f+9)/(2f+3)$. The arguments for both cases are very similar so we present only one.

*Case I.* $\frac{\overline{\gamma}}{a} < \frac{2f+9}{2f+3}$.

Let platform $i$ increase its quality by a small $\epsilon > 0$; platform $i$ becomes the high quality platform. Results from Theorem 1 imply that the resulting equilibrium profit $\pi_i$ for the high quality platform given the subgame $(\overline{\gamma}, a, \epsilon, y_j)$ can be expressed as follows using a Taylor series expansion, $\pi_i(\epsilon, 0) = Q(\overline{\gamma}, f, a)\epsilon - \frac{1}{2}I'(0)\epsilon - o(|\epsilon)|^2|$, where $Q(\overline{\gamma}, f, a)$ is a positive number. There exists an $\epsilon^*$ such that for all $\epsilon \in (0, \epsilon^*)$ the above quantity is positive. Thus platform $i$ would prefer to set quality $\epsilon \in (0, \epsilon^*)$ instead of 0. □

We now proceed to find the sets in which the best replies lie given each platform's investment level. Given quality choice $y_\alpha$, platform $\beta$ can choose a best reply that depends on whether it acts as a high-quality or a low-quality platform. In the former case it chooses a reply in the domain $(y_\alpha, \infty)$ and in the latter case it chooses a reply in the domain $[0, y_\alpha)$. Lemma 9 shows that if platform $\alpha$ invests in a positive quality and platform $\beta$ acts as a low quality platform, then platform $\beta$ prefers not to invest. By symmetry a similar claim exists for platform $\alpha$ given platform $\beta's$ quality choice. We now proceed to prove Theorem 2.

*Proof.* Given that platform $\beta$ invests in $y_\beta > 0$ platform $\alpha$ can choose to be a low quality or a high quality platform. The best response for platform $\alpha$ given it acts as a low-quality (high-quality) platform is given by $B_\alpha(y_\beta) = 0$ $(B_\beta(y_\beta) \in (y_\beta, \infty))$. The former follows from Lemma 9 and the latter from Lemma 10. The overall best response is the maximum of these two best responses, i.e., the value for which the profit function is highest. Let $\bar{y} = \{y_\alpha | y_\alpha > y_\beta\}$. Given $y_\beta > 0$ then $B_\alpha(y_\beta) \in \{0 \bigcup \bar{y}\}$. If $y_\beta = 0$ then the best response is given by $y^* = \{y_\alpha | I'(y_\alpha) = r'_\alpha(y_\alpha)\}$; where $r_\alpha(y_\alpha)$ is the revenue made by the high quality platform. Note that given the tuple $(\overline{\gamma}, a, f)$, $r_\alpha(y_\alpha)$ is a linear function in $y_\alpha$. It follows that $y^*$ is a singleton since the profit function is concave in $y_\alpha$. Therefore given $y_\beta = 0$, $B_\alpha(y_\beta) = y^*$. Since the explicit form of the revenue function depends on whether $\overline{\gamma}/a < (9+2f)/(3+2f)$ or $\overline{\gamma}/a \geq (9+2f)/(3+2f)$ we have two implicit characterizations of this singleton. The sets in which platform $\alpha's$ best response lies is similar by symmetry. Consequently, the only points of intersection are $[y^*, 0]$ and $[0, y^*]$. □

### A.6. Proof of Theorem 3

*Proof.* We show that for a $c \geq 1$ and $f$ large enough the pair $(y^*, 0)$, as defined in the Theorem statement is a SPE. Some of the expressions involved are too large to put in the paper. Where this is the case we state the importance of the results for the proof. In Theorem 2, we showed that the pair $(0, y^*)$ is a candidate equilibrium pair. In particular $y^*$ is the best response of one platform given the other platform chooses not to invest. We proceed to show that for a quadratic investment function when one platform invests in $y^*$ the other opts not to invest concluding that a SPE exists for this investment function. To analyze this response we partition the space in which $\overline{\gamma}/a$ lies into three regions corresponding to the types of market configurations that exits in each of the region. Note that the revenue function, which equals profit plus investment cost, is of a different form in each of these regions, hence the different analysis.

*Case I.* $1 < \frac{\overline{\gamma}}{a} < \frac{5f+18}{5f+6}$*:*

If the platform acts as a high quality platform, i.e., chooses $y > y^*$ there are three possible revenue functions that may result depending on the choice of $y$. Let $r(y, y^*)$ denote the revenue function of the platform that is responding to an investment level of $y^*$ by the other platform. This revenue function is made up of a concatenation of three other revenue functions. These are,

$$
\begin{aligned}
&r^{ci}(y, y^*) \text{ if } y \in \left(y^*, y^* \frac{1}{2}\frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}\right) \\
&r^{cc}(y, y^*) \text{ if } y \in \left[y^* \frac{1}{2}\frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}, y^* \frac{1}{2}\frac{(9-2f)\overline{\gamma}+(2f+3)a}{(f+15)a-(f+9)\overline{\gamma}}\right] \\
&r^{ui}(y, y^*) \text{ if } y \in \left(y^* \frac{1}{2}\frac{(9-2f)\overline{\gamma}+(2f+3)a}{(f+15)a-(f+9)\overline{\gamma}}, \infty\right)
\end{aligned}
$$

The restrictions over which these functions are defined are derived from the market configurations in Theorem 6. The first refers to the revenue function when the market is covered with an interior solution, the second refers to the revenue function when the market is covered with a corner solution and the last refers to when the market is uncovered with masses present in both configurations.

We find a differentiable upper-bound of $r(y,y^*)$ and show that the best response when the platform acts as a high-quality platform, under this function, is dominated by the best response when it acts as a low-quality platform. Let this upper-bound be denoted by $r^{\text{est}}$. Lemma 11 shows that $r^{ci}(y,y^*)$ over the domain $y>y^*$ is an upper-bound of $r(y,y^*)$. So we find the best response under this function and compare it with the best response when the platform acts as a low-quality platform and opts not to invest. Let the maximum profit value when the platform acts as a high-quality platform under the upper-bound revenue be denoted by $\pi^{\text{est}}(y^{\text{est}},y^*)$, where

$$\begin{aligned} y^{\text{est}} =& \text{argmax } \pi^{\text{est}}(y,y^*), \\ & \text{s.t. } y>y^*. \end{aligned} \tag{42}$$

Let the maximum profit value when the platform acts as a low-quality platform be denoted by $\pi^{\text{low}}$. From Lemma 9 this occurs at $y=0$. One can show that $\pi^{\text{low}}>\pi^{\text{est}}$.[21]

*Case II.* $\frac{5f+18}{5f+6} \le \frac{\overline{\gamma}}{a} < \frac{9+2f}{9+2f}$*:*

In this region two market configurations are possible if the platform picks $y>y^*$. These are a preempted market and a covered market with a corner solution. Since in the preempted market several price equilibria exist we pick the one that yields the highest price and use that to calculate an upper-bound for the profit function. We denote it by $\pi^{\text{est}}$. We compare this solution against $\pi^{\text{low}}$, which is the profit of the platform when it chooses to be the low-quality platform. In a similar manner to the first case we show $\pi^{\text{low}}>\pi^{\text{est}}$. In particular $\pi^{\text{est}}-\pi^{\text{low}} = -2/27f^2(3a-4f\overline{\gamma}-3\overline{\gamma})(a-\overline{\gamma}+f\overline{\gamma})<0$ whenever $f>1-a/k$.

*Case III.* $\frac{5f+18}{5f+6} < \frac{\overline{\gamma}}{a}$

In this interval, when the platform decides to act as a high quality platform, only the pre-empted market exists. Moreover, there are multiple price equilibria. So we use the price equilibria that yields the highest possible profit and use it to derive an upper-bound for the profit function. The analysis then proceeds in exactly the same manner as that in case II because the upper-bound for the profit function when the platform chooses to act as a high-quality is the same as that in case II.

□

Lemma 11. *If* $y \in \left[y^* \frac{1}{2}\frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}, \infty\right]$ *then* $r^{ci}(y,y^*) \ge r(y,y^*)$.

*Proof.* Note that $r(y,y^*)$ is continuous since $\lim_{y\to\bar{y}} r^{ci}(y,y^*) = r^{cc}(\bar{y},y^*)$ and $\lim_{y\to\hat{y}} r^{cc}(y,y^*) = r^{ui}(\hat{y},y^*)$ where $\bar{y}=y^*\frac{1}{2}\frac{(21-2f)\overline{\gamma}+(2f-9)a}{(f+9)a-(f-3)\overline{\gamma}}$ and $\hat{y}=y^*\frac{1}{2}\frac{(9-2f)\overline{\gamma}+(2f+3)a}{(f+15)a-(f+9)\overline{\gamma}}$. The revenue functions $r^{cc}(y,y^*)$ and $r^{ui}(y,y^*)$ are increasing in $y$ since the derivatives are positive. Moreover, the former is convex in $y$, while the later is concave $y$. The difference $r'^{ci}(\bar{y},y^*)-r'^{cc}(\bar{y},y^*)$[22] is positive whenever $1<\frac{\overline{\gamma}}{a}<\frac{f+15}{f+9}$. Furthermore, $r'^{ci}(\hat{y},y^*)-$

[21] The expression showing that the difference of the two terms is positive has many terms and is omitted for the sake of clarity.

[22] $r'^{ci}(\bar{y},y^*)-r'^{cc}(\bar{y},y^*) = f((f+9)a-(f+3)k)((5f+6)k+(18-5f)a)/(486a)$.

$r'^{ui}(\hat{y},y^*)^{23}$ and $r'^{ui}(\hat{y},y^*) - r'^{cc}(\hat{y},y^*)^{24}$ are also positive in this interval. This coupled with the fact that the revenue functions are increasing implies that $r^{ci}(y,y^*) > r^{cc}(y,y^*)$ in the domain of $y$ where $r^{cc}(y,y^*)$ is defined. Moreover, the concavity of $r^{ui}(y,y^*)$ also implies that $r^{ci}(y,y^*) > r^{ui}(y,y^*)$ in the domain of $y$ where the latter is defined. Therefore, whenever $1 < \frac{\overline{\gamma}}{a} < \frac{f+15}{f+9}$ then $r^{ci}(y,y^*) \geq r(y,y^*)$. In the case where $\frac{f+15}{f+9} < \frac{\overline{\gamma}}{a} < \frac{18+5f}{6+5f}$ only two market configurations exist, a covered market with an interior solution and an uncovered market with a corner solution. The difference between the revenue functions over the region where the covered market with a corner solution is defined is concave in $y$ and increasing. This follows because i) the second derivative of the difference, $r'^{ci}(y,y^*) - r'^{cc}(y,y^*)$, is negative[25] whenever $y > y^*$; ii) the difference is increasing in $y$ since the derivative is positive in this range; iii) for $y > \bar{y}$, the range in which the covered market with corner solution is defined, the difference is positive. □

## Appendix B: Technical Details for the Non-neutral Model

### B.1. Sets of Prices that yield relations (i),(ii) and (iii) as defined in section (5.2)

We now define sets $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$ and $\mathcal{W}_{R(iii)}$ that contain price pairs $(w_\alpha, w_\beta)$ that may yield relations $(i)$, $(ii)$ and $(iii)$, as defined in sections 5.2, respectively. First we define set $\mathcal{W}_{R(i)}$ by solving for the price pairs for which $F_i(y_\alpha,\cdot) > F_i(y_\beta,\cdot)$. We characterize this set below;

$$\mathcal{W}_{R(i)} = \begin{cases} \{(w_\alpha,w_\beta)|w_\alpha < \overline{w}_\alpha, w_\beta > \overline{w}_\beta \text{ and } w_\alpha < q_\alpha f A + q_\alpha/q_\beta w_\beta\} & \text{if } y_\alpha \in \Lambda, \\ \{(w_\alpha,w_\beta)|w_\alpha < \overline{w}_\alpha, w_\beta > \overline{w}_\beta\} & \text{if } y_\alpha \in \Delta, \\ \{(w_\alpha,w_\beta)|w_\alpha \geq 0, w_\beta \geq 0\} & \text{if } y_\alpha \in \Theta. \end{cases}$$

Here, $(q_\alpha, q_\beta) = (2/3, 1/3)$, $A = (y_\alpha - y_\beta)(\overline{\gamma} + a)^2/(\overline{\gamma} - a)$, $\Lambda = \left(y_\beta, y_\beta \frac{f(\overline{\gamma}+a)^2-(\overline{\gamma}-a)^2}{f(\overline{\gamma}+a)^2-(\overline{\gamma}^2-a^2)}\right)$, $\Delta = \left[y_\beta \frac{f(\overline{\gamma}+a)^2-(\overline{\gamma}-a)^2}{f(\overline{\gamma}+a)^2-(\overline{\gamma}^2-a^2)}, 2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}\right)$, $\Theta = \left[2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}, \infty\right)$. If $y_\alpha \in \Lambda$ then $\overline{w}_\alpha = q_\alpha f A + q_\alpha(\overline{\gamma} - a) f y_\beta$, and $\overline{w}_\beta = (\overline{\gamma} + a) f y_\alpha - f A q_\beta$. If $y_\alpha \in \Delta$ then $\overline{w}_\alpha = (\overline{\gamma} + a) 2/3 f y_\alpha$ and $\overline{w}_\beta = (\overline{\gamma} - a) 1/3 f y_\beta$.

We similarly characterize set $\mathcal{W}_{R(ii)}$ by solving for price pairs for which $F_i(y_\alpha,\cdot) < F_i(y_\beta,\cdot)$;

$$\mathcal{W}_{R(ii)} = \begin{cases} \{(w_\alpha,w_\beta)|w_\alpha > \underline{w}_\alpha, w_\beta < \underline{w}_\beta \text{ and } w_\alpha > q_\alpha f A + q_\alpha/q_\beta w_\beta\} & \text{if } y_\alpha \in \Lambda, \\ \{(w_\alpha,w_\beta)|w_\alpha > \underline{w}_\alpha, w_\beta < \underline{w}_\beta\} & \text{if } y_\alpha \in \Delta, \\ \{(w_\alpha,w_\beta) \in \emptyset\} & \text{if } y_\alpha \in \Theta. \end{cases}$$

Here, $(q_\alpha, q_\beta) = (1/3, 2/3)$, $A$, $\Lambda$, $\Delta$, and $\Theta$ are as previously defined. In addition, if $y_\alpha \in \Lambda$ then $\underline{w}_\alpha = q_\alpha f A + q_\alpha(\overline{\gamma} - a) f y_\beta$, and $\underline{w}_\beta = (\overline{\gamma} + a) f y_\alpha - f A q_\beta$. If $y_\alpha \in \Delta$ then $\underline{w}_\alpha = (\overline{\gamma} + a) 1/3 f y_\alpha$ and $\underline{w}_\beta = (\overline{\gamma} - a) 2/3 f y_\beta$.

We similarly characterize set $\mathcal{W}_{R(iii)}$ by solving for price pairs for which $F_i(y_\alpha,\cdot) = F_i(y_\beta,\cdot)$;

$$\mathcal{W}_{R(iii)} = \begin{cases} \{(w_\alpha,w_\beta)\}|w_\alpha = \underline{w}_\alpha & \text{if } w_\beta \leq (\overline{\gamma} - a)1/2 f y_\beta \text{ and } y_\alpha \in \Lambda, \\ \{(w_\alpha,w_\beta)\}|w_\beta = \underline{w}_\beta & \text{if } w_\alpha \geq (\overline{\gamma} + a)1/2 f y_\beta \text{ and } y_\alpha \in \Lambda, \\ \{(w_\alpha,w_\beta)\}|w_\alpha = q_\alpha f A + q_\alpha/q_\beta w_\beta & \text{if } w_z \in ((\overline{\gamma} - a) y_z/2, (\overline{\gamma} + a) y_z/2) \\ & \quad \text{for } z \in \{\alpha,\beta\} \text{ and } y_\alpha \in \Lambda, \\ \{(w_\alpha,w_\beta)|w_\alpha \geq \underline{w}_\alpha, w_\beta \leq \underline{w}_\beta, & \text{if } y_\alpha \in \Delta \bigcup 2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}, \\ \{(w_\alpha,w_\beta) \in \emptyset, & \text{if } y_\alpha \in \Theta / 2y_\beta \frac{\overline{\gamma}}{\overline{\gamma}+a}. \end{cases}$$

[23] $r'^{ci}(\hat{y},y^*) - r'^{ui}(\hat{y},y^*) = f(a-\overline{\gamma})(5f^2a^2 + 32fa^2 - 261a^2 - 10f^2\overline{\gamma}a - 58fa\overline{\gamma} + 120\overline{\gamma}a + 5f^2\overline{\gamma}^2 + 26f\overline{\gamma}^2 - 3\overline{\gamma}^2)/(27a((f+3)a + (3-f)\overline{\gamma}))$.

[24] $r'^{ui}(\hat{y},y^*) - r'^{cc}(\hat{y},y^*) = (-5fa + 5f\overline{\gamma} + 24a)(fa + 15a - f\overline{\gamma} - 9\overline{\gamma})^2 fa((f+3)a + (3-f)\overline{\gamma})/486$.

[25] $r'^{ci}(y,y^*) - r'^{cc}(y,y^*) = -(3fy^{*2}(\overline{\gamma} - a)^2)/(8a(y - y^*)^3)$.

Here, $(q_\alpha, q_\beta) = (1/2, 1/2)$, $A$, $\Lambda$, $\Delta$, and $\Theta$ are as previously defined. In addition, if $y_\alpha \in \Lambda$ then $\underline{w}_\alpha = q_\alpha f A + q_\alpha(\overline{\gamma} - a) f y_\beta$, and $\underline{w}_\beta = (\overline{\gamma}+a) f y_\alpha - f A q_\beta$. If $y_\alpha \in \Delta$ then $\underline{w}_\alpha = (\overline{\gamma}+a)1/2 f y_\alpha$ and $\underline{w}_\beta = (\overline{\gamma}-a)1/2 f y_\beta$.

We note that if a price pair lies on the intersection of any of the sets $\mathcal{W}_{R(i)}$, $\mathcal{W}_{R(ii)}$, and $\mathcal{W}_{R(iii)}$, then more than one equilibrium allocation exists.

## B.2. Relation $(ii)$ does not hold on the equilibrium Path

We first present a lemma showing that if a price subgame results in multiple CP allocation equilibria, such that relations $(i)$ and $(ii)$ hold, the CP allocation equilibrium for which relation $(i)$ holds yields the highest profit for platform $\alpha$. Let $\overline{\pi}_\alpha(w_\alpha, w_\beta)$ $(\widehat{\pi}_\alpha(w_\alpha, w_\beta))$ denote platform $\alpha's$ profit whenever relation $(i)$ $((ii))$ holds.

Lemma 12. *Given a price pair $(w_\alpha, w_\beta)$ such that the CP allocations which yield relations $(i)$ and $(ii)$ can occur, then $\overline{\pi}_\alpha(w_\alpha, w_\beta) > \widehat{\pi}_\alpha(w_\alpha, w_\beta)$.*

*Proof.* Let $\overline{r}_\alpha, \overline{r}_\beta$ $(\widehat{r}_\alpha, \widehat{r}_\beta)$ denote the CP demand when relation $(i)$ $((ii))$ holds. It follows that $\overline{r}_\alpha \geq \widehat{r}_\alpha$ and $\overline{r}_\beta \leq \widehat{r}_\beta$ since $\overline{q}_\alpha > \widehat{q}_\alpha$ and $\overline{q}_\beta < \widehat{q}_\beta$. Moreover, $\overline{p}_\alpha \overline{q}_\alpha > \widehat{p}_\alpha \widehat{q}_\alpha$. To see this note that,

$$\begin{aligned}\overline{p}_\alpha =& \overline{q}_\alpha (y_\alpha((\overline{\gamma}+a) + (\overline{\gamma}-a)\overline{r}_\alpha) - y_\beta((\overline{\gamma}+a) + (\overline{\gamma}-a)\overline{r}_\beta)),\\ \widehat{p}_\alpha =& \widehat{q}_\alpha (y_\beta((\overline{\gamma}+a) + (\overline{\gamma}-a)\widehat{r}_\alpha) - y_\beta((\overline{\gamma}+a) + (\overline{\gamma}-a)\widehat{r}_\beta)).\end{aligned}$$

Therefore, $\overline{\pi}_\alpha(w_\alpha, w_\beta) = \overline{r}_\alpha w_\alpha + \overline{p}_\alpha \overline{q}_\alpha > \widehat{r}_\alpha w_\alpha + \widehat{p}_\alpha \widehat{q}_\alpha = \widehat{\pi}_\alpha(w_\alpha, w_\beta)$. □

We now show in the next lemma that if an SPE exists then the CP allocation that holds in the equilibrium path does not yield relation $(ii)$.

Lemma 13. *If $(w^*_\alpha, w^*_\beta)$ is an SPE of the quality subgame then the CP allocation on the equilibrium path does not yield relation $(ii)$.*

*Proof.* From Appendix B.1, there are three cases to consider; $y_\alpha \in \Lambda$, $y_\alpha \in \Delta$ and $y_\alpha \in \Theta$. In the last case, a CP allocation equilibrium that yields relation $(ii)$ does not exist. Therefore, we only consider the first two cases.

*(a)* $y_\alpha \in \Lambda$

Let $\widehat{\pi}_\alpha(w^*_\alpha, w^*_\beta) = \widehat{r}_\alpha w^*_\alpha + p^*_\alpha \widehat{q}_\alpha$ denote the revenue under the equilibrium price pair $(w^*_\alpha, w^*_\beta)$. Here, $\widehat{r}_\alpha$ $(\widehat{q}_\alpha)$ refers to the mass of CPs (consumers) at equilibrium and $p^*_\alpha$ is the price offered to the consumer at equilibrium. Let $\overline{w}_\alpha = w^*_\alpha - \epsilon$, where $\epsilon > 0$. Observe that since $y_\alpha \in \Lambda$ and $(w^*_\alpha, w^*_\beta) \in \mathcal{W}_{R(ii)}$, it is always possible to choose $\epsilon$ such that $(\overline{w}_\alpha, w^*_\beta) \in \mathcal{W}_{R(ii)}$. Let $\pi_\alpha(w_\alpha, w^*_\beta)$ be the profit function for platform $\alpha$ generated by choosing the CP allocation equilibrium which yields relation $(ii)$, whenever more than one CP allocation equilibrium is possible. This profit function is quadratic and concave in $w_\alpha$ over the interval $\mathcal{I} = (\max\{\underline{w}_\alpha, 1/3fA + 1/2w^*_\beta\}, (\overline{\gamma}-a)1/3fy_\alpha)$. Here, $\underline{w}_\alpha$ is as defined in appendix $B.1$, where $(q_\alpha, q_\beta) = (1/3f, 2/3f)$. The unrestricted maximum $w^u_\alpha = \text{argmax}\ \pi(w_\alpha, w^*_\beta) < \max\{\underline{w}_\alpha, (\overline{\gamma}-a)1/3fy_\alpha\}$. Therefore, $\pi(w_\alpha, w^*_\beta)$ is decreasing in the interval $\mathcal{I}$. If we pick an $\epsilon$ small enough, then $\overline{w}_\alpha \in \mathcal{I}$. This implies that $\pi(\overline{w}_\alpha, w^*_\beta) > \widehat{\pi}_\alpha(w^*_\alpha, w^*_\beta)$. Note also that at price $(\overline{w}_\alpha, w^*_\beta)$, the revenue arising under the CP allocation equilibrium which yields relation $(i)$ is higher, see lemma 12. Therefore price $\overline{w}_\alpha$ dominates price $w^*_\alpha$ and platform $\alpha$ has an incentive to deviate.

*(b)* $y_\alpha \in \Delta$

Let $\overline{w}_\alpha = \underline{w}_\alpha - \epsilon$, where $\epsilon > 0$ such that $(\overline{w}_\alpha, w^*_\beta) \in \mathcal{W}_{R(i)} \cap \mathcal{W}^c_{R(iii)}$. Here, $\underline{w}_\alpha$ is as defined in appendix B.1, with $(q_\alpha, q_\beta) = (1/3f, 2/3f)$. Denote the revenue under the equilibrium price pair $(w^*_\alpha, w^*_\beta)$ by $\widehat{\pi}_\alpha(w^*_\alpha, w^*_\beta) = \widehat{r}_\alpha w^*_\alpha + p^*_\alpha \widehat{q}_\alpha$. At the price pair $(\overline{w}_\alpha, w^*_\beta)$ only one CP allocation equilibrium is possible; the one that yields relation $(i)$. Let $\overline{\pi}_\alpha(\overline{w}_\alpha, w^*_\beta) = \overline{r}_\alpha \overline{w}_\alpha + \overline{p}_\alpha \overline{q}_\alpha$ represent the revenue at this price.

We next show that $\overline{\pi}_\alpha(\overline{w}_\alpha, w^*_\beta) > \widehat{\pi}_\alpha(w^*_\alpha, w^*_\beta)$. First we note that revenue made on the CP side by platform $\alpha$ is higher under the new price since $\overline{r}_\alpha > \widehat{r}_\alpha = 0$. This follows from the fact that $\overline{w}_\alpha < \underline{w}_\alpha < (\overline{\gamma} + a)2/3fy_\alpha$. Therefore, a positive mass of CPs will patronize the platform since they gain positive utility upon joining. Revenue on the consumer side is also higher under this new deviation. To see this, note that since relation $(i)$ holds, $\overline{q}_\beta < \widehat{q}_\beta$. This implies $\overline{r}_\beta < \widehat{r}_\beta$ which further implies that $\overline{p}_\alpha > p^*_\alpha$. Observe that $\overline{q}_\alpha > \widehat{q}_\alpha$ (since the CP allocation equilibrium yields relation $(i)$), $y_\alpha \geq y_\beta$ and,

$$\begin{aligned} \overline{p}_\alpha =& \overline{q}_\alpha (y_\alpha((\overline{\gamma} + a) + (\overline{\gamma} - a)\overline{r}_\alpha) - y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\overline{r}_\beta)), \\ p^*_\alpha =& \widehat{q}_\alpha (y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\widehat{r}_\alpha) - y_\beta((\overline{\gamma} + a) + (\overline{\gamma} - a)\widehat{r}_\beta)). \end{aligned}$$

□

### B.3. Proof of Theorem 4

In this Appendix, we show that given the tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta$ a unique SPE exists in the CP price game. We define the baseline CP price game as the game induced by selecting the CP allocation equilibrium which yields relation $(i)$, whenever multiple equilibria exist. We then show that this game has a unique SPE. In addition, we show that all reduced extensive form games, for which an SPE exist, have this same unique SPE. Thus without loss of generality we may only consider the baseline CP price subgame. Recall we are considering reduced extensive form games in which the CP allocation equilibria chosen, whenever multiple equilibria exist, are those that yield either relation $(i)$ or $(iii)$.

*Proof.* The proof involves the following two steps.

*Step. 1 Baseline-CP price game has a unique SPE*

Given a tuple $(\overline{\gamma}, a, f, y_\alpha, y_\beta)$ such that $y_\alpha > y_\beta$ and price $w_\beta$, we denote platform $\alpha's$ profit function by $\pi_\alpha(w_\alpha, w_\beta)$. This profit function is quadratic in $w_\alpha$ over the range $\mathcal{I} = [(\overline{\gamma} - a)2/3fy_\alpha, min\{(\overline{\gamma} + a)2/3fy_\alpha, 2/3fA + 2w_\beta\}]$. It is linear and decreasing for $w_\alpha < (\overline{\gamma} - a)2/3fy_\alpha$. Let $w^u_\alpha = \text{argmax}\ \pi_\alpha(w_\alpha, w_\beta)$ be the unrestricted maximum of the quadratic function. This value is given by $w^u_\alpha = \frac{1}{9}((3-2f)\overline{\gamma} + (3+2f)a)fy_\alpha$. Whenever $y_\alpha > y_\beta$ then $w^u_\alpha < min\{(\overline{\gamma} + a)2/3fy_\alpha, 2/3fA + 2w_\beta\}$, where $w_\beta \geq (\overline{\gamma} - a)1/3fy_\beta$. Therefore, given $w_\beta$, the best response is given by $w^*_\alpha = \max\{w^u_\alpha, (\overline{\gamma} - a)2/3fy_\alpha\}$. Given another $w'_\beta$, the profit function $\pi'_\alpha(w_\alpha, w'_\beta) = \pi_\alpha(w_\alpha, w_\beta) + k(w_\beta, w'_\beta)$ over the range $[0, 2/3fA + 2((\overline{\gamma} - a)1/3fy_\beta)]$. Therefore $w^{u\prime}_\alpha = w^u_\alpha = \text{argmax}\ \pi_\alpha(w_\alpha, w'_\beta)$ and the best response $w^*_\alpha$ is also the same. Thus given any $w_\beta$ the best response is a constant $w^*_\alpha$. We can similarly show that given any $w_\alpha$ the best response by platform $\beta$ is given by $w^*_\beta = \max\{w^u_\beta, (\overline{\gamma} - a)1/3fy_\beta\}$ where $w^u_\beta = \frac{1}{18}((3+f)\overline{\gamma} + (3-f)a)fy_\beta$. Thus the pair $(w^*_\alpha, w^*_\beta)$ form a unique SPE.

If $w^u_\alpha \leq (\overline{\gamma} - a)2/3fy_\alpha$ then all CPs will connect to platform $\alpha$. Following some algebra the former holds when $\frac{\overline{\gamma}}{a} \geq \frac{9+2f}{3+2f}$. On the other hand, only a fraction of the CPs join the platform whenever $\frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f}$. If

$w^u_\beta \leq (\overline{\gamma} - a)1/3fy_\beta)$ then all CPs will connect to platform $\beta$. Following some algebra, one can show the former holds when $\frac{\overline{\gamma}}{a} \geq \frac{9-f}{3-f}$. Therefore, only a fraction of the CPs join the platform whenever $\frac{\overline{\gamma}}{a} < \frac{9-f}{3-f}$.

We define the following sets of prices which we use to characterize market configurations that hold at the SPE.

$$\begin{aligned}
\mathcal{R}^n_I =& \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) < 1,\ r_\beta(w_\alpha, w_\beta) < 1\},\\
\mathcal{R}^n_{II} =& \{(w_\alpha, w_\beta) | r_\alpha(w_\alpha, w_\beta) < 1, r_\beta(w_\beta, w_\alpha) = 1; \text{ or } r_\alpha(w_\alpha, w_\beta) = 1, r_\beta(w_\alpha, w_\beta) < 1\},\\
\mathcal{R}^n_{III} =& \{(w_\alpha, w_\beta | r_\alpha(w_\alpha, w_\beta) = 1,\ r_\beta(w_\alpha, w_\beta) = 1)\}.
\end{aligned}$$

The set $\mathcal{R}^n_I$ consists of prices $(w_\alpha, w_\beta)$ such that only a fraction of the CPs in the market subscribe to the platforms. Set $\mathcal{R}^n_{II}$ consists of prices $(w_\alpha w_\beta)$ such that the market is covered; all CPs patronize either platform $\alpha$ or $\beta$ but not both. Lastly set $\mathcal{R}^n_{III}$ consists of a pair of prices such that the market is covered with all CPs patronizing both platforms. We summarize results of the previous paragraph below.

*a)* If $1 < \frac{\overline{\gamma}}{a} < \frac{9+2f}{3+2f}$, then $(w^*_\alpha, w^*_\beta) \in \mathcal{R}^n_I$.

*b)* If $\frac{9+2f}{3+2f} \leq \frac{\overline{\gamma}}{a} < \frac{9-f}{3-f}$ then $(w^*_\alpha, w^*_\beta) \in \mathcal{R}^n_{II}$.

*c)* If $\frac{9-f}{3-f} \leq \frac{\overline{\gamma}}{a} < \infty$ then $(w^*_\alpha, w^*_\beta) \in \mathcal{R}^n_{III}$.

*Step. 2 CP price games that have a SPE, have the same SPE as the baseline CP price game* Given a price game and any pair $(w_\beta, w'_\beta) \in \mathbb{R}^+$, the profit value $\pi_\alpha(w_\alpha, w_\beta) = \pi_\alpha(w_\alpha, w'_\beta) + k(w_\beta, w'_\beta)$ for all $w_\alpha \in [0, 2/3fA + 2((\overline{\gamma} - a)1/3fy_\beta)]$ except possibly at $w_\alpha = 1/2fA + w'_\beta$ and $w_\alpha = 1/2fA + w_\beta$. Therefore the best response for platform $\alpha$ given platform $\beta$ charges $w_\beta$ is given by $w^*_\alpha$ as defined in the previous step if $w^*_\alpha \neq 1/2fA + w_\beta$. In the case $w^*_\alpha = 1/2fA + w_\beta$ then a best response does not exist. Similarly, we can show that the best response given any $w_\alpha$ is given by $w^*_\beta$, as defined in *step 1*, if a best response exists. Thus if the best responses intersect they only do so at the price pair $(w^*_\alpha, w^*_\beta)$. □

### B.4. The case when $y_\alpha = y_\beta$

Lemma 13 implies that neither relation $(i)$ or $(ii)$ hold at the SPE when $y_\alpha = y_\beta$. Therefore, if an SPE exists it must be that relation $(iii)$ holds. We bound the maximum revenue value that can result in instances for which an SPE exists. Given a tuple $(\overline{\gamma}, a, f)$, we show later in the investment stage that this pair is not an SPE because either platform has an incentive to deviate.

We now provide an upper-bound for the revenue gained by the platforms if an SPE exists. Revenue for both platforms is derived only from the CP side. This follows because only relation $(iii)$ can hold in equilibrium; due to Bertrand competition, platforms earn no revenue from the consumer side. As discussed in section 5.3, the allocation of consumers is evenly divided when relation $(iii)$ holds, i.e, $q_\alpha = q_\beta = 1/2f$. Moreover, if relation $(iii)$ holds then $r_\alpha = r_\beta$ which further implies $w_\alpha = w_\beta$. Let revenue for platform $\alpha$ and $\beta$ be represented by $\pi_\alpha(w^*_\alpha, w^*_\beta)$ and $\pi_\beta(w^*_\alpha, w^*_\beta)$ respectively at the SPE price $(w^*_\alpha, w^*_\beta)$. Then $\pi_\alpha(w^*_\alpha, w^*_\beta) = \pi_\beta(w^*_\alpha, w^*_\beta)$ where $w^*_\alpha \in ((\overline{\gamma} - a)(q_\alpha y_\alpha), (\overline{\gamma} + a)(q_\alpha y_\alpha))$. We consider only prices in this range because other prices are dominated and will not be picked in equilibrium. Consider the function $\pi(w_\alpha) = r_\alpha w_\alpha$ where $r_\alpha = 1/2a(\overline{\gamma} + a - w_\alpha/q_\alpha y_\alpha)$. This function is concave and quadratic in $w_\alpha$ and at the value $w^*_\alpha$ we have $\pi(w^*_\alpha) = \pi_\alpha(w^*_\alpha, w^*_\beta)$. Let,

$$\hat{w}_\alpha = \text{argmax } \pi(w_\alpha),$$

$$\text{s.t. } w_\alpha \in ((\overline{\gamma}-a)(q_\alpha y_\alpha), (\overline{\gamma}+a)(q_\alpha y_\alpha)).$$

If an SPE exists platform $\alpha' s$ profits are bounded by $\pi(\hat{w}_\alpha)$. Since platform $\beta' s$ profits are the same as $\alpha' s$ they are also bounded from above by the same value.

### B.5. Proof of Theorem 5.

Given a quality choice $y_\beta$, we derive platform's $\alpha$ best response and vice versa. We then find the intersection points that form the SPE. We will give Lemmas that define the best responses and then we will be able to infer the SPE's from these responses.

LEMMA 14. *Let $\mathcal{R}_1$ hold. Then*

$$B_i(y_\beta) = \begin{cases} y^*(\overline{\gamma}, a, f, c) & \text{if } y_\beta < \overline{y}, \\ 0 & \text{if } y_\beta \geq \overline{y}, \end{cases}$$

*where*

$$y^*(\overline{\gamma}, a, f, c) = \frac{a^2 f}{216c}\left(\frac{(2f+3)^2}{a^3}\overline{\gamma}^2 + \frac{90-8(f-3)^2}{a^2}\overline{\gamma} + \frac{((2f+9)^2-36)}{a}\right),$$
$$\overline{y}(\overline{\gamma}, a, f, c) = \frac{48\overline{\gamma}fa + 36fa^2 + 12f\overline{\gamma}^2 + 4f^2\overline{\gamma}^2 - 8f^2\overline{\gamma}a + 4f^2a^2 + 9\overline{\gamma}^2 + 18\overline{\gamma}a + 9a^2)^2}{-432ac(-60\overline{\gamma}a - 45a^2 - 15\overline{\gamma}^2 + 2fa^2 + 2f\overline{\gamma}^2 - 4\overline{\gamma}fa)}.$$

*Proof.* We first find the best response given $y_\beta = 0$. Let $y^* = \text{argmax } \pi_\alpha(y_\alpha, y_\beta), \text{s.t. } y_\alpha \in \mathbb{R}_+$. In region $\mathcal{R}_1$ the market is uncovered as shown in Appendix B.3. The profit function $\pi_\alpha(y_\alpha, y_\beta)$ is quadratic and concave in $y_\alpha$ in this market configuration. The best response, $B_\alpha(y_\beta) = y^*(\cdot)$ exists since $\pi_\alpha(y_\alpha, y_\beta)$ is coercive and its value is that given in the Lemma statement.

Next, we find the best response when $y_\beta > 0$. The price equilibria that holds depends on whether platform $\alpha$ acts as the high-quality or the low-quality platform. Indeed, given $y_\beta > 0$ platform $\alpha' s$ choice of investment $y_\alpha$ will determine which of the following three relations defined in section 5.2 will hold on the equilibrium path.

If $y_\alpha$ is higher (lower) than $y_\beta$ then relation $(i)((ii))$ will hold. When $y_\alpha = y_\beta$ , relation (iii) may hold if an SPE exists. Therefore, we partition the domain $[0, \infty)$ depending on whether platform $\alpha$ acts a high quality or low quality platform. These partitions are defined as, $I_1 = [0, y_\beta)$, $I_2 = (y_\beta, \infty)$. In interval $I_1$ $(I_2)$ only $(ii)((i))$ holds. In contrast, at the point $y_\alpha = y_\beta$ relation (iii) holds if an SPE exists. In order to find the best reply given $y_\beta$ we proceed as follows. We find the best response of platform $\alpha$ in partitions $I_1$ and $I_2$. We pick the best reply among these choices and show it dominates the maximum possible choice given $y_\alpha = y_\beta$ as calculated in the previous appendix.

$$\begin{aligned} y^*_{\alpha 1} =& \text{argmax } \hat{\pi}_\alpha(y_\alpha, y_\beta) \\ & \text{s.t. } y_{\alpha 1} \in I_1. \end{aligned} \tag{43}$$

Where $\hat{\pi}_\alpha$ is the profit function in the interval $I_1$. In a like manner we denote the best reply in interval $I_2$ by $y^*_{\alpha 2}$. Formally,

$$\begin{aligned} y^*_{\alpha 2} =& \text{argmax } \pi_\alpha(y_\alpha, y_\beta) \\ & \text{s.t. } y_{\alpha 2} \in I_2. \end{aligned} \tag{44}$$

Where $\pi_\alpha$ is the profit function in the interval $I_2$. The profit function $\hat{\pi}_\alpha$ is concave in $y_\alpha$. Let $y^{ur}_{\alpha 1}$ be the unrestricted solution. This value is less than zero in region $\mathcal{R}_1$. Therefore, the lower constraint in the maximization problem 43 binds and we have $y^*_{\alpha 1} = 0$. On the other hand , the unrestricted maximization of problem 44 yields $y^*_{\alpha 2} = y^*(\overline{\gamma}, a, f, c)$ as defined in the statement of the Lemma.

Next we compare the profit values at the solutions in both intervals. Let $\pi^*_\alpha = \pi_\alpha(y_\alpha, y_\beta)|_{y_\alpha = y^*_{\alpha 2}}$ and $\hat{\pi}^*_\alpha = \hat{\pi}_\alpha(y_\alpha, y_\beta)|_{y_\alpha = 0}$. One can show that the difference $\pi^*_\alpha - \hat{\pi}^*_\alpha$ is decreasing in $y_\beta$. Moreover, the two are equal at $y_\beta = \overline{y}$; the value presented in the Lemma statement. Hence whenever $y_\beta < \overline{y}$, $y^*_{\alpha 2}$ dominates $y^*_{\alpha 1}$ and vice versa. In addition, $y^*_{\alpha 2} > y_\beta$ whenever $y_\beta < \overline{y}$. Therefore this solution lies in the interior of $I_2$.

To complete this proof we now show that the profit attainable when $y_\alpha = y_\beta$ is less than $\max\{\hat{\pi}^*_\alpha, \pi^*_\alpha\}$. We denote the upper-bound profit value when $y_\alpha = y_\beta$ by $\overline{\pi}^*_\alpha$. It suffices to show that this value is less than $\pi^*_\alpha$ when $y_\beta > \overline{y}$ and less than $\hat{\pi}^*_\alpha$ when $y_\beta < \overline{y}$. We first show the former, i.e, the difference $\pi^*_\alpha - \overline{\pi}^*_\alpha$ is positive whenever$y_\beta > \overline{y}$. This difference[26] is convex and quadratic in $y_\beta$. Moreover it has roots at 0 and $\overline{y}$. Therefore the difference is positive whenever $y_\beta > \overline{y}$. Next we show $\hat{\pi}^*_\alpha - \overline{\pi}^*_\alpha$ is positive. The difference is quadratic and convex in $y_\beta$. Moreover, the roots are imaginary[27] thus we infer that the difference is positive. □

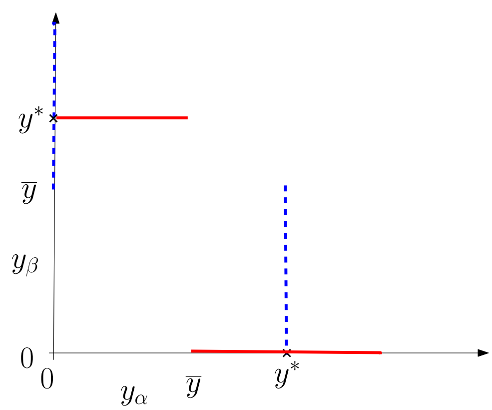


**Figure 13 The best reply responses of both platforms in region $\mathcal{R}_1$. The intersection points, $(y^*, 0)$ and $(0, y^*)$, give the equilibrium investment levels in region $\mathcal{R}_1$.**

A similar analysis follows for platform $\beta$. The best responses of the platforms intersect at points $(y^*, 0)$ and $(0, y^*)$. Consequently this points form an SPE, see Figure 13.

We next state a number of Lemmas that yield the other results in Thereof 5. We omit the proofs because they are very similar. Where there's significant diversion we add comments.

Lemma 15. *Let Assumption 1, $f > \frac{1}{3}$ and $\mathcal{R}_2$ hold. Then*

$$B_i(y_\beta) = \begin{cases} y^*_h(\overline{\gamma}, a, f, c) & \text{if } y_\beta < \overline{y}, \\ y^*_l(\overline{\gamma}, a, f, c) & \text{if } y_\beta \geq \overline{y}, \end{cases}$$ [28]

[26] $y_\beta - \overline{y} = y_\beta(48f^2\overline{\gamma}a + 36f^2a^2 + 12f^2\overline{\gamma}^2 + 8f^3\overline{\gamma}^2 - 16f^3\overline{\gamma}a + 8f^3a^2 - 27f\overline{\gamma}^2 - 54\overline{\gamma}fa - 27fa^2 + 432cy_\beta a)/432a$.

[27] $\text{root} = (27a^2 - 16f^2a^2 + 144fa^2 + 32f^2\overline{\gamma}a + 48f\overline{\gamma}^2 + 192\overline{\gamma}fa - 16f^2\overline{\gamma}^2 + 27\overline{\gamma}^2 + 54\overline{\gamma}a \pm \sqrt{-3(\overline{\gamma}+a)(96fa+21a+21\overline{\gamma}+32f\overline{\gamma})(32f^2a^2+9a^2-64f^2\overline{\gamma}a+18\overline{\gamma}a+32f^2\overline{\gamma}^2+9\overline{\gamma}^2)})f/(864ca)$.

*where*

$$y_h^*(\overline{\gamma},a,f,c) = \frac{a^2 f}{216c}\left(\frac{(2f+3)^2}{a^3}\overline{\gamma}^2 + \frac{90-8(f-3)^2}{a^2}\overline{\gamma} + \frac{((2f+9)^2-36)}{a}\right),$$
$$y_l^*(\overline{\gamma},a,f,c) = \frac{a^2 f}{432c}\left(\frac{(f-3)^2}{a^3}\overline{\gamma}^2 - \frac{-90+2(f+6)^2}{a^2}\overline{\gamma} + \frac{((f-9)^2-72)}{a}\right).$$

Lemma 16. *Let Assumption 1 and $\mathcal{R}_3$ hold. Then*

$$B_i(y_\beta) = \begin{cases} y^*(\overline{\gamma},a,f,c) & \text{if } y_\beta < \overline{y}, \\ 0 & \text{if } y_\beta \geq \overline{y}. \end{cases}$$

*where*

$$y_h^*(\overline{\gamma},a,f,c) = \frac{f}{9c}\left(4f\overline{\gamma}^2 + 3\overline{\gamma} - 3a\right),$$
$$y_l^*(\overline{\gamma},a,f,c) = 0,$$
$$\overline{y}(\overline{\gamma},a,f,c) = \frac{(-4f\overline{\gamma} - 3\overline{\gamma} + 3a)^2 a}{c3(18\overline{\gamma}a + 9a^2 + 3\overline{\gamma}^2 - f\overline{\gamma}^2 + 2f\overline{\gamma}a - fa^2)}.$$

Lemma 17. *Let Assumption 1 and $\mathcal{R}_4$ hold. Then*

$$B_i(y_\beta) = \begin{cases} y_h^*(\overline{\gamma},a,f,c) & \text{if } y_\beta < \overline{y}, \\ y_l^*(\overline{\gamma},a,f,c) & \text{if } y_\beta \geq \overline{y}, \end{cases}$$ [29]

*where*

$$y_h^*(\overline{\gamma},a,f,c) = \frac{f}{9c}\left(4f\overline{\gamma}^2 + 3\overline{\gamma} - 3a\right),$$
$$y_l^*(\overline{\gamma},a,f,c) = \frac{af}{432c}\left(\frac{(f-3)^2}{a^3}\overline{\gamma}^2 - \frac{-90+2(f+6)^2}{a^2}\overline{\gamma} + \frac{((f-9)^2-72)}{a}\right).$$

Lemma 18. *Let $f > 0.47$ and Assumption 1 and $\mathcal{R}_5$ hold. Then*

$$B_i(y_\beta) = \begin{cases} y_h^*(\overline{\gamma},a,f,c) & \text{if } y_\beta < \overline{y}, \\ y_l^*(\overline{\gamma},a,f,c) & \text{if } y_\beta \geq \overline{y}, \end{cases}$$

*where*

$$y_h^*(\overline{\gamma},a,f,c) = \frac{f}{9c}\left(4f\overline{\gamma} + 3\overline{\gamma} - 3a\right),$$
$$y_l^*(\overline{\gamma},a,f,c) = \frac{f}{18c}\left(-2f\overline{\gamma} + 3\overline{\gamma} - 3a\right),$$
$$\overline{y}(\overline{\gamma},a,f,c) = \frac{1}{120\overline{\gamma}c}(20f^2\overline{\gamma}^2 + 36f\overline{\gamma}^2 - 36\overline{\gamma}fa + 9\overline{\gamma}^2 - 18\overline{\gamma}a + 9a^2).$$

*Proof.* We first find the best response given $y_\beta = 0$. Similar to the proof of Lemma 14 we let $y^* = \operatorname{argmax}\, \pi_\alpha(y_\alpha, y_\beta), \text{s.t. } y_\alpha \in \mathbb{R}^+$. In region $\mathcal{R}_5$ the market is covered as shown in Appendix B.3. The profit function $\pi_\alpha(y_\alpha, y_\beta)$ is quadratic and concave in $y_\alpha$ in this market configuration. The best response, $B_\alpha(y_\beta) = y^*(\cdot)$ exists since $\pi_\alpha(y_\alpha, y_\beta)$ is coercive and the solution is that given by $y_h^*(\overline{\gamma},a,f,c)$ in the Lemma statement.

Next we find the best response when $y_\beta > 0$. Given $y_\beta > 0$ a platform $\alpha$ decides to have a quality that is the same as platform $\beta$ or to be either the high or low quality platform. The choice made will determine which of the three relations defined in section 5.2 will hold.If $y_\alpha > (<) y_\beta$ then relation (i)((ii)) results. If $y_\alpha = y_\beta$

and an SPE exists then relation $(iii)$ holds. Therefore we partition the domain $[0, y_\alpha)$ into two regions that do not include the point $y_\beta$. These partitions are defined as $I_1 = [0, y_\beta)$,$I_2 = (y_\beta, \infty)$.

In order to find the best reply we proceed as follows. We find the best response of platform $\alpha$ in partitions $I_1$ and $I_2$. We pick the best reply among these choices and show it dominates the choice $y_\alpha = y_\beta$. Let

$$y^*_{\alpha 1} = \text{argmax } \hat{\pi}_\alpha(y_\alpha, y_\beta) \quad (45)$$
$$\text{s.t. } y_{\alpha 1} \in I_1.$$

Here, $\hat{\pi}_\alpha$ is the profit function in the interval $I_1$. In a like manner we denote the best reply in interval $I_2$ by $y^*_{\alpha 2}$. Formally,

$$y^*_{\alpha 2} = \text{argmax } \pi_\alpha(y_\alpha, y_\beta) \quad (46)$$
$$\text{s.t. } y_{\alpha 2} \in I_2.$$

Here, $\pi_\alpha$ is the profit function in the interval $I_2$. The profit function $\hat{\pi}_\alpha$ is concave in $y_\alpha$. Let $y^*_{\alpha 1}$ be the unrestricted solution of problem 45. Its value is given by $y^*_{\alpha 1} = y^*_l(\overline{\gamma}, a, f, c)$. This value is greater than zero in region $\mathcal{R}_5$. The unrestricted maximization of problem 46 yields $y^*_{\alpha 2} = y^*_h(\overline{\gamma}, a, f, c)$ as defined in the statement of the Lemma.

Next we compare the profit values at these solutions. Let $\pi^*_\alpha = \pi_\alpha(y_\alpha, y_\beta)|_{y_\alpha = y^*_{\alpha 2}}$ and $\hat{\pi}^*_\alpha = \hat{\pi}_\alpha(y_\alpha, y_\beta)|_{y_\alpha = y^*_{\alpha 1}}$. One can show that the difference $\pi^*_\alpha - \hat{\pi}^*_\alpha$ is decreasing in $y_\beta$. Hence whenever $y_\beta < \overline{y}$, $y^*_{\alpha 2}$ dominates $y^*_{\alpha 1}$ and vice versa. Moreover, the two are equal at $y_\beta = \overline{y}$; this is the value presented in the Lemma statement. In addition, one can show that $\overline{y} \in (y^*_{\alpha 1}, y^*_{\alpha 2})$ whenever $f > 3(2\sqrt{19} - 1)/50 \approx 0.47$.

To complete this proof we now show that the highest profit attainable when $y_\alpha = y_\beta$, and a SPE in the quality subgame exists, is less than $\max\{\hat{\pi}^*_\alpha, \pi^*_\alpha\}$. We denote the upper-bound profit value when $y_\alpha = y_\beta$ by $\overline{\pi}^*_\alpha$. It suffices to show that this value is less than $\pi^*_\alpha$. From previous Appendix it follows that $\overline{\pi}^*_\alpha < \pi^*_\alpha$. To see this note that when $y_\alpha = y_\beta$, and an SPE results, platform $\alpha$ makes no revenue on the consumer side. $\square$

The best responses for platform $\beta$ are similarly derived. These responses intersect at the investment pairs $(y^*_h, y^*_l)$ and $(y^*_l, y^*_h)$. Thus these form the SPE as stated in the Theorem, see Figure 14.

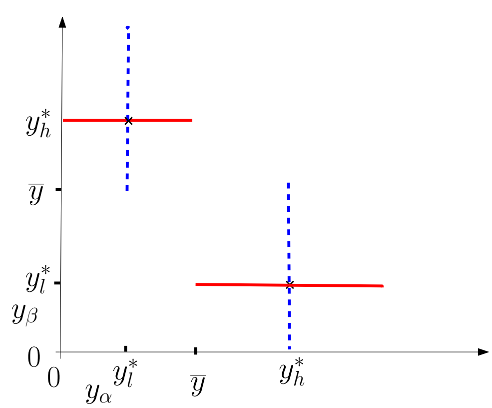


**Figure 14 The best reply responses of both platforms in region $\mathcal{R}_1$. The intersection points, $(y^*, 0)$ and $(0, y^*)$, give the equilibrium investment levels in region $\mathcal{R}_1$.**

## Appendix C: Social Welfare, CP and Consumer Surplus Comparison

### C.1. Social Welfare Comparison

In this section we provide comparison of social welfare at the SPE under both models. First, we characterize the difference between welfare of the non-neutral and neutral regime in terms of the following exogenous parameters $(\overline{\gamma}, a, f, c)$. We show that this difference is non-negative and that in general, the non-neutral regime is favored to the neutral regime. Since the SPE for both models have been characterized for $f > 3/5$ the comparisons are also based for the same range of $f$.

We denote the difference between the non-neutral and neutral welfare by $dw$. The welfare functions at the SPE's have different forms in both models depending on whether the market is covered or uncovered. The restrictions on the tuple $(\overline{\gamma}, a, f, c)$ that define the limits in which the CP market is uncovered and covered coincide in both models. So we compare the welfare between the two regimes for each of the regions defined in Section 5.5. After some algebra, the difference in welfare for the different regions are given below:

| Regions | Difference in welfare $dw$ |
|---|---|
| $\mathcal{R}_1$ and $\mathcal{R}_3$ | $dw = 0$ |
| $\mathcal{R}_2$ and $\mathcal{R}_4$ | $dw = \frac{((\overline{\gamma}-a)^2 f^2 - 6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + 9(\overline{\gamma}+a)^2)(-(\overline{\gamma}-a)^2 f^2 + (60a^2 - (\overline{\gamma}-7a)^2)f + 6(\overline{\gamma}+a)(2\overline{\gamma}+3))f^2}{93312ca^2}$ |
| $\mathcal{R}_5$ | $dw = \frac{1}{324c} f^2 (3a + 5\overline{\gamma} + 2\overline{\gamma} f)(3\overline{\gamma} - 3a - 2\overline{\gamma} f)$ |

For $f \geq 3/5$ the value $dw$ is positive in regions $\mathcal{R}_2$, $\mathcal{R}_4$ and $\mathcal{R}_5$. In these regions, the low-quality platform makes an investment that increases the gross value of CPs and Consumer surplus compared to their values in the neutral regime. On the other hand, in regions $\mathcal{R}_1$ and $\mathcal{R}_3$ the welfare in both regimes is the same because the investment levels are the same.

### C.2. CP surplus comparison

In this section we compare the CP surplus in both regimes. Let $dcp$ denote the difference in CP surplus between the two regimes. The following table shows this difference in the regions defined in Section 5.5.

| Regions | Difference in welfare $dcp$ |
|---|---|
| $\mathcal{R}_1$ and $\mathcal{R}_3$ | $dcp = 0$ |
| $\mathcal{R}_2$ and $\mathcal{R}_4$ | $dcp = \frac{f^2(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + (\overline{\gamma}-a)^2 f^2 + 9(\overline{\gamma}+a)^2) \times (3(\overline{\gamma}+a) - f(\overline{\gamma}-a))^2}{186624ca^2}$ |
| $\mathcal{R}_5$ | $dcp = \frac{1}{54c} f^2 (\overline{\gamma}(3 - 2f) - 3a)a$ |

For regions $\mathcal{R}_1$ and $\mathcal{R}_3$ the CP surplus is the same in both regimes. In these regions the investments across both platforms are the same. Therefore, the aggregate utility gained by the CPs is the same across both regimes. In regions $\mathcal{R}_2$, $\mathcal{R}_4$ and $\mathcal{R}_5$ the value $dcp$ is positive.

### C.3. Consumer surplus Comparison

In this subsection we compare the consumer surplus in both regimes. Let $dc$ denote the difference in consumer surplus between the two regimes. The following table shows the consumer surplus difference in the regions defined in Section 5.5.

| Regions | Difference in welfare $dc$ |
|---|---|
| $\mathcal{R}_1$ and $\mathcal{R}_3$ | $dc = 0$ |
| $\mathcal{R}_2$ and $\mathcal{R}_4$ | $dc = \frac{f^2(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f+(\overline{\gamma}-a)^2f^2+9(\overline{\gamma}+a)^2)(1+10f)(3(k+a)(k+3a)-(k-a)^2f)f^2}{93312ca^2}$ |
| $\mathcal{R}_5$ | $dc = \frac{1}{162c}f^2(\overline{\gamma}(3-2f)-3a)(\overline{\gamma})(1+10f)$ |

In regions $\mathcal{R}_1$ and $\mathcal{R}_3$ consumer surplus is the same under both regimes because both platforms invest in the same qualities. However, in regions $\mathcal{R}_2$, $\mathcal{R}_4$ and $\mathcal{R}_5$ the value $dc$ is positive.

### C.4. Platform profits Comparison

In this subsection we compare aggregate and individual platform profits in both regimes. Let $dtp$ denote the difference in aggregate platform profit between the two regimes. The following table shows this difference in the regions defined in Section 5.5.

| Regions | Difference in aggregate $dtp$ |
|---|---|
| $\mathcal{R}_1$ and $\mathcal{R}_3$ | $dtp = 0$ |
| $\mathcal{R}_2$ and $\mathcal{R}_4$ | $dtp = \frac{f^2(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f+(\overline{\gamma}-a)^2f^2+9(\overline{\gamma}+a)^2)(-54(k+a)(k+3a)f+17(k-a)^2f+9(k+a)^2)}{186624ca^2}$ |
| $\mathcal{R}_5$ | $dtp = -\frac{1}{108c}f^2(\overline{\gamma}(3-2f)-3a)(\overline{\gamma})(k(6f-1)+a).$ |

In regions $\mathcal{R}_1$ and $\mathcal{R}_3$ the profits of the platforms are the same in both the neutral and non-neutral regimes. This follows because the investments across platforms are equal in both regimes. In regions $\mathcal{R}_2$, $\mathcal{R}_4$ and $\mathcal{R}_5$ the aggregate profit in the neutral regime is higher than that in the non-neutral regime since $dtp$ is positive.

Next we show that profit of the high-quality platform is larger in the neutral regime in regions $\mathcal{R}_2$, $\mathcal{R}_4$ and $\mathcal{R}_5$. Let $dtph$ be the difference between the high-quality platform's profit in both regimes.

| Regions | Difference in profits $dtph$ |
|---|---|
| $\mathcal{R}_1$ and $\mathcal{R}_3$ | $dtph = 0$ |
| $\mathcal{R}_2$ and $\mathcal{R}_4$ | $dtph = \frac{f^3(-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f+(\overline{\gamma}-a)^2f^2+9(\overline{\gamma}+a)^2)(-3(k+a)(k+3a)f+17(k-a)^2+9(k+a)^2f)}{11664ca^2}$ |
| $\mathcal{R}_5$ | $dtph = -\frac{4}{81c}\overline{\gamma}f^3(\overline{\gamma}(3-2f)-3a)$ |

Given the restrictions that define regions $\mathcal{R}_2$, $\mathcal{R}_4$ and $\mathcal{R}_5$, *dtph* is positive whenever $f > 3/5$.

Next we show that in general the low-quality platform prefers the non-neutral regime. Let the difference of profit between the low-quality platform in the neutral and non-neutral regimes be *dtpl*. The table below shows this quantity in the different regions.

| Regions | Difference in profits *dtpl* |
|---|---|
| $\mathcal{R}_1$ and $\mathcal{R}_3$ | $dtpl = 0$ |
| $\mathcal{R}_2$ and $\mathcal{R}_4$ | $dtpl = \frac{1}{186624ca^2} f^2 \left((-6(\overline{\gamma}+a)(\overline{\gamma}+3a)f + (\overline{\gamma}-a)^2 f^2 + 9(\overline{\gamma}+a)^2)\right)^2$ |
| $\mathcal{R}_5$ | $dtpl = \frac{1}{324c} f^2 \overline{\gamma} f^3 (\overline{\gamma}(3-2f) - 3a)^2$ |

The value *dtpl* is non-negative. In regions $\mathcal{R}_1$ and $\mathcal{R}_3$ the profits are the same because the investments across both platforms are the same. In contrast, the low-quality platform's profits in regions $\mathcal{R}_2$, $\mathcal{R}_4$ and $\mathcal{R}_5$ are superior in the non-neutral regime because *dtpl* is positive.