Competition in Electricity Markets with Renewable Energy Sources*

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First draft: June 2014
This draft: May 2015

Abstract

This paper studies the effects of the diversification of energy portfolios on the merit order effect in an oligopolistic energy market. The merit order effect describes the negative impact of renewable energy, typically supplied at the low marginal cost, to the electricity market. We show when thermal generators have a diverse energy portfolio, meaning that they also control some or all of the renewable supplies, they offset the price declines due to the merit order effect because they strategically reduce their conventional energy supplies when renewable supply is high. In particular, when all renewable supply generates profits for only thermal power generators this offset is complete — meaning that the merit order effect is totally neutralized. As a consequence, diversified energy portfolios may be welfare reducing. These results are robust to the presence of forward contracts and incomplete information (with or without correlated types). We further use our full model with incomplete information to study the volatility of energy prices in the presence of intermittent and uncertain renewable supplies.

Keywords: Merit order effect, imperfect competition, geographic proximity, diversification, oligopoly pricing, intermittent sources

JEL Classification: D6, D62.

*We thank seminar participants at MIT, UC Berkeley and the 2014 Allerton Conference for helpful comments.

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1 Introduction

With mounting concerns over climate change caused by fossil fuels, there has been growing reliance on renewable energy.[1] Currently, 67 countries, including all EU countries, have renewable energy policy targets, mandating electricity companies to provide a minimal fraction of total electricity supply from renewables. In the United States, for example, this target is set to grow to 20% by 2020, while it is 33% in the UK and also 20% in the European Union. These targets have motivated many conventional (thermal) energy companies to seek a diversified energy portfolio and increase their investments in renewable supply. The European energy giant Alstom has thus concluded: "A diverse energy portfolio is the only sound business and policy strategy able to address any Energy & Climate scenario" (Lalwani and Khoo [2013]). Though the high setup costs of renewable plants are often subsidized by public funds (U.S. Energy Information Administration [2013]), they are argued to benefit the economy not just by reducing fossil fuel emissions but also by delivering cheaper energy to consumers through the *merit order effect (MoE)*. The merit order effect arises because renewable energy has negligible marginal costs and reduces the spot equilibrium price as illustrated in Figure 1.[2] Figure 2 depicts the merit order effect in action in the German market and shows the strong negative correlation between the supply of (intermittent) renewable energy and the wholesale electricity price.[3]

This paper argues that the objective of a diversified energy portfolio of conventional energy companies conflicts with the presumed benefits from increasing supply of renewables in terms of lower prices (via the merit order effect). We show that when the increase in the supply of renewable energy takes the form of diversified energy portfolios, the MoE is partially neutralized. In the extreme case where all of the supply of renewables comes in the form of such diversified energy portfolios (meaning that it is supplied by the same conventional energy companies), the MoE is fully neutralized, and greater renewable supplies, or more favorable realization of renewable supply outcomes, have no impact on equilibrium energy prices.

Our baseline and simplest model uses the standard Cournot oligopolistic competition setup to establish the partial or full neutralization of the MoE. The main economic force leading to this result is that diversified producers have an incentive to offset the price declines due to the MoE by reducing their conventional energy supplies. The greater is the

[1] For the climate and environmental benefits of renewable energy, see for example Dincer [2000].

[2] In addition, renewable energy also enjoys priority dispatch due to regulation.

[3] The merit order effect is also documented in detail in several other seen in several electricity markets (e.g. German market (Sensfuss et al. [2008], Nicolosi and Fursch [2009], Cludiusa et al. [2014]), German-Austrian market (Wurzburg et al. [2013]), Danish market (Munksgaarda and Morthorstb [2008]), Irish market (O'Mahoney and Eleanor [2011]), and Australian market (McConnella et al. [2013])).

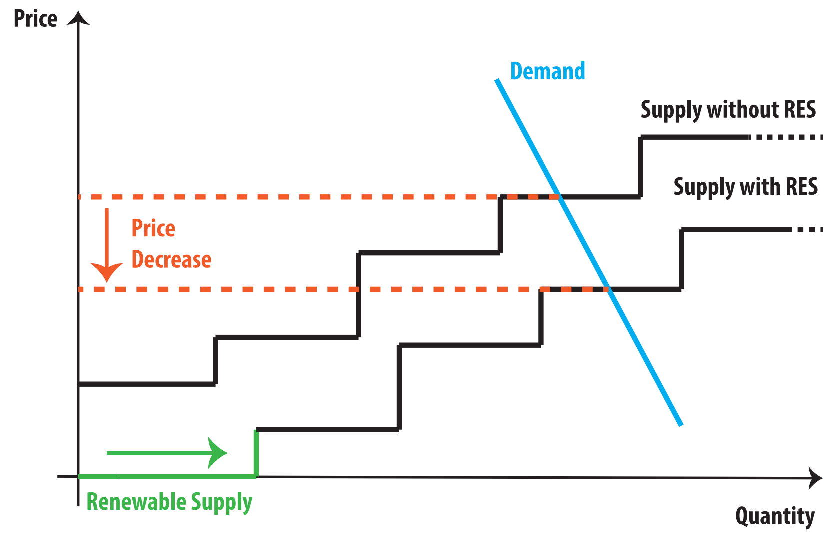

Figure 1: The merit order effect on the equilibrium (market) prices.

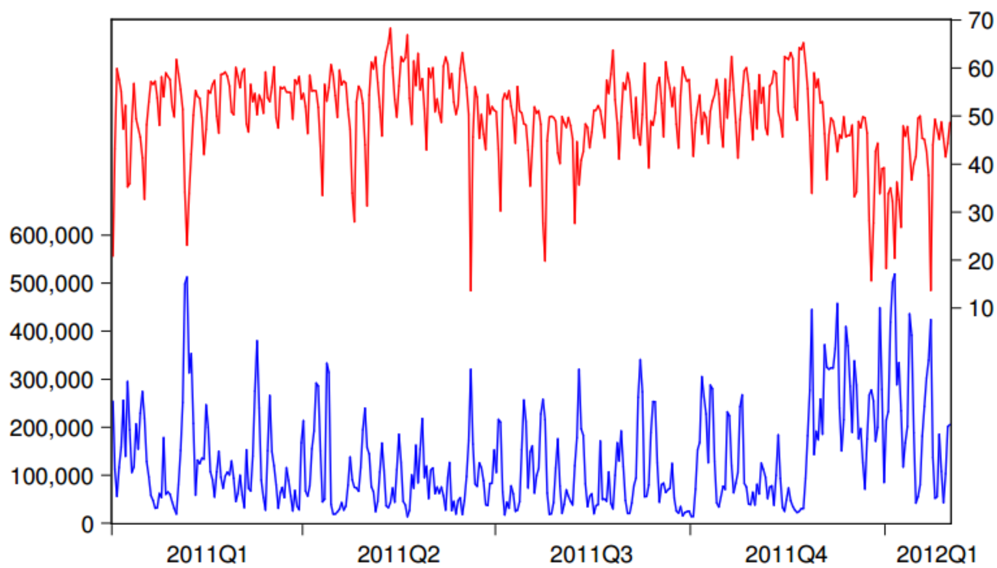

Figure 2: The relationship between prices in red ($/MWh) and renewable energy output in blue (MWh) in Germany. Source: European Energy Exchange (EEX).

supply of renewables, the stronger is this incentive. This force is related to the strategic substitutes property of Cournot competition, inducing suppliers to cut production when the supply of renewables is high. But crucially, this incentive is exacerbated because diversified energy companies take into account the loss of profits from their own renewable supplies that would result in the absence of such a cut.

We then enrich this baseline model to incorporate two important features of energy markets: forward contracts, and correlated imperfectly observed shocks to geographically proximate renewable energy suppliers. Forward contracts play a central role in many energy markets, both because of private parties' incentives to hedge pricing risk and because of regulations mandating forward contracts for generators.[4] We analyze forward contracts by adapting the seminal work of Allaz and Vila [1993], who demonstrated that forward contracts lead to lower equilibrium prices in oligopolistic markets. We show that our main results on the partial and full neutralization of the MoE applies in the presence of forward contracts in exactly the same fashion as in our baseline model.

Our full model incorporates both forward contracts and correlated, imperfectly observed shocks. We model this latter feature by assuming that renewable energy supplies in similar geographic areas are subject to locally correlated variability, and that each firm only observes its own realization of renewable supply, but is aware of the structure of local correlation. We are not aware of any other work in the literature developing a tractable model of incomplete information Cournot equilibrium with forward contracts and renewable energy. The incomplete information Cournot equilibrium in this case again shows the neutralization of the MoE. In addition, this version of the model enables us to study the implications of renewable supply on price volatility. Focusing on spatial configurations in which the correlation of renewable supply decays according to distance across plants, we show that the spot price volatility increases as the distance among renewable plants increases. This is because when renewable plants are far apart so that there is less correlation among renewable supply, they create more miscoordination in supplies, increasing price volatility. Using this intuition, we further show that among *all* geographic configurations with "regular" structures, the maximum price volatility occurs when renewable plants have a "ring" structure and the minimum price volatility arises in geographic configurations exhibiting a "complete" structure.

Finally, we study the profit and welfare implications of diversified renewable portfolios. Intuitively, diversified energy portfolios are beneficial for thermal producers, but

[4] This is because of concerns that without forward contracts, the generators (with market power) may manipulate energy prices (e.g., Borenstein [2002]). Empirical research also indicates the extent of forward contracting by generators has been an important determinant of the competitive performance of several markets (e.g. Australian market Wolak [2000], Texas market Hortaçsu and Puller [2008]).

detrimental for consumers. Most importantly, the negative effect on consumers resulting from higher markups dominates, and overall welfare declines with greater diversification.

Literature This work is related to the literature on energy market and oligopoly pricing. Generally, there are two ways of modeling energy markets (Joskow [2001], Borenstein [2002]): pooling market and bilateral trades. In a pooling market, all the energy producers sell their produced energy to a centrally operated pool, and then all the energy customers buy from the pool. In contrast, in a bilateral trade producers and customers deal separately and then inform the independent system operator that they have agreed to a transaction.

The main approaches to run pooling markets are based on the supply function equilibrium (SFE) model and the Cournot competition. In the SFE each producer submits a supply function to a system operator, who will set a uniform market clearing price, as a result producers compete both in quantity and price. This model was first developed by Klemperer and Meyer [1989], and later applied in the wholesale electricity markets by Green and Newbery [1992], Rudkevich et al. [1998], Baldick and Hogan [2002], Baldick et al. [2004]. In the Cournot model, instead of a supply function, each producer simply bids its desired production amount, and the market price is determined to clear the market given consumer demand. The Cournot setup is a good approximation to several energy markets, including California's electricity industry (Borenstein et al. [1995], Borenstein and Bushnell [1999], Borenstein et al. [1999]), New Zealand's electricity markets (Scott and Read [1996]), and congestion pricing in transmission networks (Hogan [1997], Oren [1997]).[5]

Ventosa et al. [2005] and Willems et al. [2009] have detailed discussions about the pros and cons of Cournot and SFE models. In particular, Willems et al. [2009] have tested both models using the dataset of Germany's electricity market. Their results indicate the calibrated SFE and Cournot models perform almost equally well, and on the basis of this, these authors suggest the use of Cournot models for short-term analysis and the SFE models for long-term analysis.

There is also a literature in engineering, studying the introduction of renewable supply in electricity markets (e.g. Meyn et al. [2010], Varaiya et al. [2011], Bitar et al. [2012], Rajagopal et al. [2012], Nair et al. [2014]). Meyn et al. [2010] study an electricity market equilibrium in the presence of renewable supply, and Bitar et al. [2012] analyze the optimal bidding strategies for renewable producers in the real time market. Kim and Powell

[5] See also Yao et al. [2008], Downward et al. [2010], Bose et al. [2014] for other applications of the Cournot model in electricity markets.

[2011] study energy commitments made by renewable producers in presence of electrical storage, while Korpaas et al. [2003] study scheduling and operation of storage for renewable energy producers in wholesale electricity markets. Varaiya et al. [2011], Rajagopal et al. [2012], Nair et al. [2014] study long-term contracts in the presence of renewable energy supplies. None of these works consider optimal pricing in electricity markets with renewable energy sources in the presence of forward contracts and (potentially correlated) incomplete information, nor do they discuss the neutralization of the MoE.

The work most closely related to ours is the recent independent paper by Ben-Moshe and Rubin, which also discusses the implications of diversified portfolios on the MoE. However, in their model MoE is not always present and their results are developed under the special case of linear inverse demand and quadratic costs (and without endogenous forward contracts or incomplete information). Most importantly, this paper does not contain our results on the full neutralization of the MoE and on the negative welfare effects of diversified energy portfolios.

Finally, our analysis of forward contracts is related to the growing literature building on Allaz and Vila [1993]. Hughes and Kao [1997] demonstrate the importance of the public knowledge of the forward commitment. Green [1999] studies the role of forward contracts with linear supply functions, while Mahenc and Salanie [2004] focus on forward contracts in differentiated Bertrand competition in the spot market, the ability to sign forward contracts can reduce competition. Ferreira [2003] examines a context in which there are infinite forward contracting rounds and demonstrates that a kind of "folk-theorem" result can arise, supporting a range of equilibria. Liski and Montero [2006] demonstrate conditions in which repeated contracting can facilitate tacit collusion. Green and Coq [2010] argue that the risk of facilitating collusion is greatly reduced when the contracts are of longer term (i.e., cover several periods).

The rest of the paper is organized as follows. Section 2 presents general description of the model. Section 3 studies the effects of forward contracts. Section 4 presents the multi-stage oligopoly version of the model under presence of both forward contracts and incomplete information and correlated shocks capturing geographic proximity of renewable plants. Section 4.1 solves the model for a unique equilibrium. Sections 4.2 and A.2 study price volatility. We conclude in Section 5. Derivations, proofs and extra results are found in Appendices.

2 Model

We start by analyzing the impact of diversification on the merit order effect without forward contracts and assuming complete information. Forward contracts and our full model incorporating both forward contracts and incomplete information will be introduced in the subsequent sections.

2.1 General Description

We consider an oligopolistic energy market consisting of $n \geq 2$ producers that have a diverse energy portfolio, i.e., each producer can supply energy both from conventional thermal generators (that use gas or fuel) and renewable plants. More specifically, each producer i owns a generator that produces q_i units of thermal energy at cost $C(q_i)$, where C is a convex and differentiable function. In addition to thermal energy, the economy also has a total of R units of renewable energy available at zero marginal cost. We assume that each producer owns a fraction δ/n of this supply, where $\delta \in [0,1]$. Let $Q = \sum_{i=1}^{n} q_i$ denote the total amount of thermal energy produced by the generators. The inverse demand (specifying the market price as a function of total supply) is given by $P(Q+R)$, where P is a differentiable function.

Producers compete a la Cournot by choosing their thermal energy supply q_i (they do not have a supply decision regarding renewable energy) to maximize their profits given by

$$\Pi_i = (q_i + \delta R/n)P(Q+R) - C(q_i).$$

We will look for a Cournot-Nash equilibrium of this game and refer to it as *equilibrium* for short.

In the next theorem, we present one of the main results of our paper, providing both some basic properties of the unique equilibrium and analyzing the effects of a more diversified portfolio for the producers.

Theorem 1 (**Merit Order Effect and Diversification**). *(i) Assume that the inverse demand function P is strictly decreasing and concave. Then there exists a unique equilibrium such that the following hold:*

- *The equilibrium price p^* is a nonincreasing function of the total renewable supply, i.e., $\frac{\partial p^*}{\partial R} \leq 0$ (an effect referred to as the merit order effect (MoE)).*
- *The equilibrium price is strictly increasing in δ, i.e., $\frac{\partial p^*}{\partial \delta} > 0$. That is, the equilibrium markup is increasing in the extent of diversification, and the MoE is partially*

neutralized due to diversification.

(ii) (Full Neutralization of the MoE): *The MoE is* fully neutralized *if and only if producers are fully diversified and the cost function is linear. That is,* $\frac{\partial p^*}{\partial R} = 0$ *if and only if* $\delta = 1$ *and* C *is linear.*

The first part of the theorem shows that the well-known merit order effect is present in our model and a greater supply of renewable energy reduces the market price. This can be seen as follows: a greater supply of renewable energy available at zero marginal cost has priority dispatch in supplying the demand and thus translates into a reduced residual demand to be fulfilled by thermal generators. Since this shifts prices along the supply curve, referred to as the merit order curve in energy economics literature, this effect is known as the merit order effect. The rest of the theorem shows that as producers become more diversified (meaning that they control a higher share of renewable energy), the merit order effect is weakened. Perhaps surprisingly, in the case when there is complete diversification (in particular, all of the available supply of renewable energy is owned by producers supplying thermal energy, captured by the parameter δ) and the cost function is linear, then the merit order effect is completely neutralized. In this case, an increase in supply of renewable energy has no impact on the market price.

The intuition for these results is instructive. As the degree of diversification increases, producers have an incentive to hold back their supply of thermal energy because they partially internalize the reduction in profits that this will cause from renewables. When diversification is full, this internalization becomes complete and every unit increase in renewable supply causes a unit decrease in the equilibrium supply of thermal energy leaving total quantity of energy in the market fixed and thus completely neutralizing the merit order effect.

2.2 Linear Economy

We next illustrate the results of Theorem 1 by focusing on an economy with linear cost and linear inverse demand.[6] In particular, we assume the cost of production of thermal energy is given by $C(q_i) = \gamma q_i$, $i = 1, \cdots, n$, where $\gamma > 0$ is a scalar, and inverse demand function is given by $P(Q + R) = \alpha - \beta(Q + R)$ where $\alpha > 0$ and $\beta > 0$ are scalars.[7]

[6] Linear cost and linear demand are adopted as a good approximation to energy markets in several previous works, e.g. Allaz and Vila [1993], Bushnell [2007], Banal-Estanol and Micola [2009].

[7] Throughout, we take α to be sufficiently large to ensure interior solution to the profit-maximization problem of oligopolists.

This allows us to provide an explicit characterization of the equilibrium supply of producers and equilibrium price highlighting the dependence on total renewable energy R and diversification δ.

Theorem 2. *The equilibrium supply of producer i is given by*

$$q_i^* = \frac{1}{(n+1)\beta}(\alpha - \gamma - \beta(R + \delta R/n)),$$

total equilibrium supply is

$$Q^* = \frac{n(\alpha - \gamma) - \beta(\delta R + nR)}{\beta(n+1)},$$

and the equilibrium price satisfies

$$p^* = \frac{1}{n+1}(\alpha + \beta(-R + \delta R) + n\gamma). \tag{1}$$

This result illustrates the main lessons from Theorem 1. For $\delta < 1$, the equilibrium price p^* is a decreasing function of R consistent with merit order effect. However $\frac{\partial^2 p^*}{\partial \delta \partial R} > 0$ and $\frac{\partial p^*}{\partial \delta} > 0$, highlighting that greater diversification increases prices by dulling the merit order effect. Moreover, when $\delta = 1$, we can also see that $\frac{\partial p^*}{\partial R} = 0$. At the same time, $\frac{\partial Q^*}{\partial R} = -1$, so that total thermal energy supply decreases one for one with the supply of renewables. This shows the full neutralization of merit order effect. This also explains why the linear cost is necessary for the full neutralization effect. Absent linear cost, even at $\delta = 1$, the supply of thermal energy does not decrease one for one with renewable supply.

In the rest of the paper, in order to facilitate our study of forward contracts and incomplete information, we will focus on the linear economy introduced in this subsection.

2.3 Welfare

We have so far seen that diversified energy portfolios neutralize the merit order effect and increase energy prices. Does this imply that such diversification can be welfare reducing? In this subsection, we show that diversified energy portfolios are indeed welfare-reducing. To simplify the exposition, we again focus on the linear economy (though the results in this subsection generalize straightforwardly to nonlinear demands and cost structures as we note below).

We define welfare in the usual fashion, as the sum of consumer and producer surplus.

In the current context, this implies that it consists of the profits of thermal producers, the profits of renewable producers and consumer surplus.[8] Thus,

$$\text{Welfare} \equiv \mathcal{W} = \underbrace{(Q+\delta R)p - \gamma Q}_{\text{(total) Thermal producers surplus}} + \underbrace{p(1-\delta)R}_{\text{Renewable producers surplus}} + \underbrace{\frac{(\alpha-p)^2}{2\beta}}_{\text{Consumer surplus}}.$$

The next theorem shows that greater diversification of energy portfolios always reduces equilibrium.

Theorem 3. *$\mathcal{W}$ is decreasing in δ. That is, greater diversification leads to lower welfare.*

Intuitively, diversified energy portfolios are beneficial for thermal producers (as we show in the Appendix E.2), but detrimental for consumers.[9] Since in this Cournot market, price is always above marginal cost, the negative effect on consumers resulting from higher markups dominates, and overall welfare declines with greater diversification.

3 Forward Contracts

As noted in the Introduction, forward contracts often play a central role in energy markets. A natural question is therefore whether our neutralization effects are robust to the presence of forward contracts. In this section, we show that the full neutralization of merit order effect continues to hold with forward contracts.

Consider an economy with two dates $t = 1, 2$ and timing of events as follows:

• **Date $t = 1$ (Contracting stage)**: At date $t = 1$, each producer i commits to a forward contract (q_i^f, p_i^f) and promises to generate a quantity q_i^f of thermal energy at price p_i^f for delivery at date $t = 2$.

[8] In particular, a linear inverse demand equivalently corresponds to consumer utility of the form $U(q) = \alpha q - \frac{\beta}{2}q^2$, and thus generates a consumer surplus of $U(q) - pq = \frac{(\alpha-p)^2}{2\beta}$.

[9] This result also holds when we focus on equilibrium welfare relative to welfare in the corresponding competitive equilibrium. This relative welfare measure leads to identical results in the linear economy, since competitor welfare is independent of δ.

The negative welfare result can be further extended to an economy with an arbitrary concave and downward inverse demand function, i.e., $P' < 0, P'' \leq 0$, and convex and increasing cost function, i.e. $C' > 0, C'' > 0$, though in this case is more natural to use the relative welfare measure. See Appendix A.1.

In Appendix E.2 we also show the profit consequences of increasing renewables for diversified thermal producers (i.e. $\delta > 0$) crucially depends on extent of δ. When thermal producers have a low share from renewables, i.e. δ is small, their profit decreases in R. However, increasing R is beneficial for therm, if their share from renewables is sufficiently high, i.e. δ is large.

• **Date** $t = 2$ **(Production stage)**: After these contracts are signed and observed by all parties, at $t = 2$, producers make a decision on the amount of their thermal energy supply (and renewable supply of R is brought into the market).

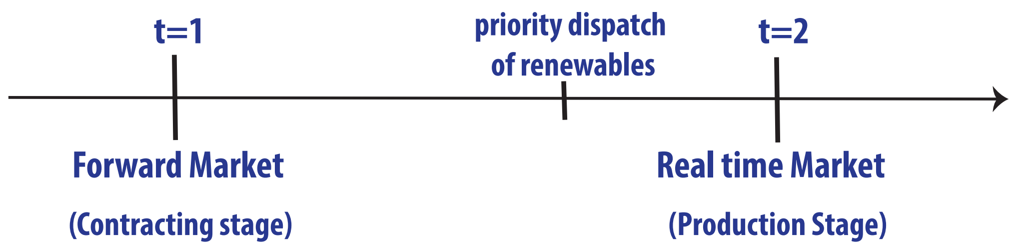

Figure 3: Timing of events.

Producer i's profit can be written as

$$\Pi_i = p(q_i - q_i^f + \delta R/n) + p_i^f q_i^f - \gamma q_i,$$

where $p = \alpha - \beta(Q + R)$. Given the multistage nature of this game, we now look for a subgame perfect Cournot-Nash equilibrium. In particular, taking as given the forward contracts signed at $t = 1$, at $t = 2$, we look for a Cournot-Nash equilibrium among the producers, and forward contracting decisions are made as a Nash equilibrium anticipating the subsequence Cournot stage.

As observed by Allaz and Vila [1993], the characterization of the equilibrium is simplified by the observation that $p_i^f = p^*$, where p^* again denotes the second stage equilibrium price. This intuition for this can be gained from a no arbitrage reasoning. If $p_i^f < p^*$, then any agent could make arbitrary profits by buying in the forward market and selling in the real time market, or the converse if $p_i^f > p^*$. With this observation, the profit function of producer i simplifies to

$$\Pi_i = (\alpha - \beta(Q + R))(q_i - q_i^f + \delta R/n) - \gamma q_i.$$

It is then straightforward to compute the Cournot-Nash equilibrium given forward contracts summarized by $q^f = (q_i^f)_{i=1,\ldots,n}$ as

$$q_i^*(q^f) = \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta\left[\sum_{j\neq i} q_j^f - nq_i^f + R + \delta R/n\right]\right).$$

Then, by backward induction, the (Cournot-Nash) equilibrium in the contract stage can be determined as the fixed point of the best response correspondences characterized by

$$q_i^{*f} \in \arg\max_{q_i^f \geq 0} \left\{ p\Big(q_i^*(q_i^f, q_{-i}^{*f}) + \delta R/n\Big) - \gamma q_i^*(q_i^f, q_{-i}^{*f}) \right\}$$
$$\text{s.t. } p = \alpha - \beta(Q^*(q_i^f, q_{-i}^{*f}) + R),$$

where $Q^*(q_i^f, q_{-i}^{*f})$ is the total supply given the forward contracts (q_i^f, q_{-i}^{*f}).

Theorem 4. *There exists a unique subgame perfect Cournot-NashNash equilibrium for the game with forward contracts. The equilibrium precommitted supply (forward quantity) of producer i is given by*

$$q_i^{*f} = \frac{n-1}{(n^2+1)\beta}\Big(\alpha + \beta(-R + \delta R) - \gamma\Big), \tag{2}$$

the equilibrium supply of producer i is given by

$$q_i^*(q_i^{*f}) = q_i^* = \frac{n}{(n^2+1)\beta}\left(\alpha + \beta\left(-R - \frac{\delta R}{n^2}\right) - \gamma\right), \tag{3}$$

and the equilibrium price satisfies

$$p^* = \frac{1}{n^2+1}\Big(\alpha + n^2\gamma + \beta(-R + \delta R)\Big). \tag{4}$$

The full neutralization of the merit order effect holds in this model, i.e., when $\delta = 1$, $\frac{\partial p^*}{\partial R} = 0$.

There are three results to highlight in this theorem. First, in the presence of forward contracts, the equilibrium price is lower compared to the case without forward contracts (in particular, p^* in Eq. (4) is strictly lower than p^* in Eq. (1)). This is because, as in Allaz and Vila [1993], forward contracts make each Cournot oligopolist act as a Stackelberg leader. In particular, each producer, by choosing to precommit to a quantity through a forward contract, forces the other producer to cut back its production in the production stage. Hence, for any value of R, the equilibrium price declines because of this additional competition effect of forward contracts. Second, for $\delta < 1$, greater R reduces the forward precommitments (i.e., q_i^{*f} in Eq. (2)). This is because, in equilibrium, forward contracts are made at the equilibrium spot price, greater renewable energy supply reduces the spot price and the precommitments. This intuition also explains that q_i^{*f} is increasing in δ. A greater δ by partially neutralizing the merit order effect increases the equilibrium spot price, and through that channel, forward precommitments. Third, for the same intuition as in Theorem 2, when $\delta = 1$, the merit order effect is fully neutralized despite presence of forward contracts.

4 Correlated Shocks and Incomplete Information

We have so far assumed that the amount of renewable energy is known both at the contracting stage and at the production stage, and we have also abstracted from the imperfect correlation in renewable supplies across different sites (e.g., across windfarms located in neighboring regions). In practice, there is considerable uncertainty about the extent of available renewable energy at any point in time, but it is generally recognized that the supplies are also correlated across various localities. Motivated by this, in this section, we consider our full model, which is an incomplete information competition setup where each producer chooses its thermal energy supply knowing its own available renewable energy but without knowing the realizations of other renewable supplies in the economy (and also still continues to enter into forward contracts). In particular, we modify our setup in the previous section by assuming that each producer owns δ fraction of a renewable plant located in its region with random amount of available energy given by R_i. We assume a general correlation structure among R_i's capturing the fact that available renewable energy has both a local and a global component (e.g., wind availability will be correlated in two nearby wind farms and less so for farms further away).

Formally, we assume each producer i privately observes the available renewable energy R_i at local plant l_i. We assume $R_i = R/n + \theta_i$, where R is a constant, and θ_i is normally distributed with mean zero and variance σ^2, i.e., $\theta_i \sim \mathcal{N}(0, \sigma^2)$. The vector $\theta = (\theta_1, \theta_2, \cdots, \theta_n)$ is assumed to be jointly normal with covariance matrix given by $\Sigma = [\text{cov}(\theta_i, \theta_j)]_{i,j=1,\dots,n}$ and $\text{cov}(\theta_i, \theta_j) = \kappa_{i,j}\sigma^2$, for $j \neq i$, where $\kappa_{i,j} \in [0, 1]$. The scalar $\kappa_{i,j}$ captures the correlation between available renewable energy at plants l_i and l_j (for instance due to correlation between wind availability at different plants). We analyze the effects of different correlation structures in Section 4.2.

Remark 1. *Our covariance matrix allows a general structure of correlation between the shocks θ_i, in particular, including both* common value *and* private value *information structures. When $\kappa_{i,j} = 1$, the θ_i parameters are perfectly correlated and we are in a common value model. When $0 < \kappa_{i,j} = \kappa < 1$, we are in a private value model. When $\kappa_{i,j} = \kappa = 0$ the parameters are independent and we are in an independent value model.*[10]

The timing of events is the same as before except that uncertainty is realized after the contracting stage at $t = 1$ and before the production stage at $t = 2$, as shown in Figure 4.

[10] A special case of this information structure (when $\kappa_{i,j} = \kappa$ for all i, j) is used by Vives [2011]. Allowing for unequal $\kappa_{i,j}$'s is important in our setting since it permits the presence of asymmetric equilibria, which is what we characterize as the correlation structure (network interactions) are not necessarily symmetric in our model.

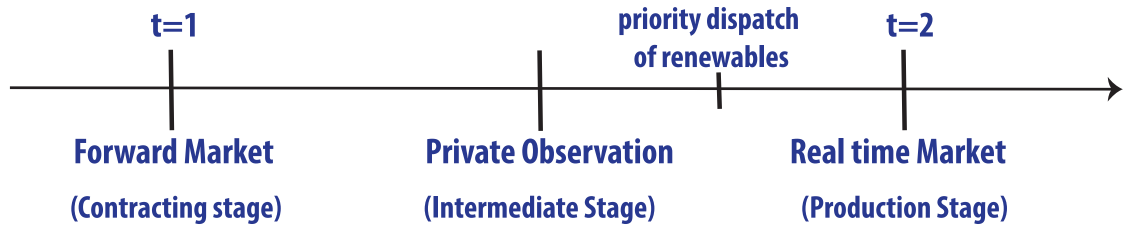

Figure 4: Timing of events.

As in the previous section, the no-arbitrage condition will imply that forward contracts have to be at the expected market price, i.e., $p_i^f = E_\theta[p]$, $i = 1, 2, \cdots, n$, where E_X is the expectation operator with respect to the random variable X. Hence, producer i's (ex-post) profit is given by

$$\Pi_i = p\left(q_i - q_i^f + \delta R_i\right) + q_i^f E_\theta[p] - \gamma q_i,$$

where $p = \alpha - \beta(\sum_{i=1}^n q_i + \sum_{i=1}^n R_i)$.

4.1 Equilibrium

Given the incomplete and multistage nature of this game, we focus on a perfect Bayesian equilibrium in linear strategies. In particular, producers choose the forward contracts at $t = 1$ anticipating the production decisions at $t = 2$ and without knowledge of the parameters θ_i. At $t = 2$, given the forward contracts, each producer chooses its quantity knowing the realization of his own θ_i, but without knowing the other θ_j's. Hence equilibrium forward quantity and supply is given by the fixed point of the below best response correspondences:

$$q_i^*(q_i^f, \theta_i) = \arg\max_{q_i} E_{\theta_{-i}}[\Pi_i | \theta_i], \ \forall\, i,$$
$$q_i^{*f} = \arg\max_{q_i^f} E_\theta[\Pi_i], \ \forall\, i.$$

The next theorem is another one of the main results of the paper and provides a full characterization of the perfect Bayesian equilibrium in this case.

Theorem 5. *There exists a unique (pure strategy) perfect Bayesian equilibrium for the incomplete information game with forward contracts. The equilibrium forward quantity of producer i is given* *by*

$$q_i^{*f} = \frac{n-1}{(n^2+1)\beta}\Big(\alpha + \beta(-R+\delta R) - \gamma\Big) \tag{5}$$

and the equilibrium supply of producer i (as a function of the parameter θ_i) is given by

$$q_i^*(\theta_i) = \frac{n}{(n^2+1)\beta}\left(\alpha + \beta\left(-R - \frac{\delta R}{n^2}\right) - \gamma\right) - a_i\theta_i \tag{6}$$

where

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \mathbf{1} + (\delta - 1)\left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1},$$

where **1** *and* **I** *denote the vector of all ones and the identity matrix, respectively. Moreover the expected value of the equilibrium price is given by*

$$E[p^*] = \frac{1}{n^2+1}\Big(\alpha + n^2\gamma + \beta(-R+\delta R)\Big).$$

The full neutralization of the merit order effect holds in this model, i.e., when $\delta = 1$, $\frac{\partial E[p^*]}{\partial R} = 0$.

Remark 2. *It follows from the preceding characterization of the equilibrium that in the* independent value model *where $\kappa_{i,j} = 0$ for all $i \neq j$, $a_1 = a_2 = \cdots = a_n = \frac{1+\delta}{2}$ and, in the* common value model *where $\kappa_{i,j} = \kappa \in (0,1)$ for all $i \neq j$, $a_1 = a_2 = \cdots = a_n = \frac{1+\delta+(n-1)\kappa}{2+(n-1)\kappa}$.*

Even though the structure of equilibria becomes richer in the presence of incomplete information (and we will discuss some of the properties of this equilibrium in more detail below), the results on the impact of the renewable supply and the extent of diversification of producers mimics Theorems 2 and 4. When $\delta < 1$, a greater supply of renewables reduces expected equilibrium price due to merit order effect (and for the same reason as in Theorem 4, it reduces q^{f*}), and greater δ increases expected equilibrium price (and tends to increase q^{f*}). In addition, when $\delta = 1$, the neutralization of the merit order effect is complete and expected equilibrium price is independent of R.

We can also observe the impact of private information θ_i on the quantity choices of producers. In particular, a higher θ_i reduces the quantity supplied because producer i itself has access to greater renewables and given the correlation across θ_i's, also expects others to have greater renewables creating another force towards lower supply through the strategic substitutes effect in Cournot competition. It is the combination of these forces that make the coefficient in front of θ_i, a_i, depend on both δ (the extent of diversification) and the correlation structure of θ_i's as in Eq. (6).

Example (duopoly with forward contracts and incomplete information) To highlight the effect of the correlation among the parameters θ_i on equilibrium, we focus on the special case with two producers. We assume $\mathrm{Cov}(\theta_1, \theta_2) = \kappa\sigma^2$, where κ is a parameter that scales inversely with the distance among the renewable plants of the producers. Using Theorem 5, we have a unique perfect Bayesian Nash equilibrium in linear strategies. The equilibrium supply of producer i is given by

$$\begin{aligned} q_i^*(\theta_i) &= \frac{2}{5\beta}\left(\alpha + \beta(-R - \delta R/4) - \gamma\right) - \left(\frac{1+\delta+\kappa}{2+\kappa}\right)\theta_i \\ &= \tilde{q}_i^* - \left(\frac{1+\delta+\kappa}{2+\kappa}\right)\theta_i, \end{aligned} \tag{7}$$

where $\tilde{q}_i^*$ is the equilibrium of the same economy with complete information. The equilibrium price satisfies

$$E[p^*] = \frac{1}{5}(\alpha + \beta(-R + \delta R) + 4\gamma).$$

Intuitively, similar to the previous cases, at equilibrium, each producer cuts back on its thermal supply in response to an increase in the renewable supply (given by θ_i) due to the strategic substitutes effect in Cournot competition. This reduction is modulated since renewable energy availability is now correlated among different plants (i.e., when my renewable energy is high, so is my competitor's) and this creates incentive for greater holding back.[11] Most interestingly, when $\delta = 1$, this modulation disappears and production becomes independent of κ. This is again an implication of the neutralization of merit order effect. When $\delta = 1$, total production of each producer i (thermal+renewable) is independent of the parameter θ_i. Since his competitor's production is independent of his private information, the correlation between their θ's does not impact the producer's supply decision.

4.2 Price volatility

The next proposition provides a characterization of price volatility and highlights its dependence on renewable share of producers δ and the correlation structure among parameters θ_i.

[11] This is because $\frac{1+\delta+\kappa}{2+\kappa}$ is increasing in κ.

Proposition 1. *The equilibrium price volatility is given by*

$$Var(p^*) = \beta^2(\delta - 1)^2 \ \boldsymbol{b}^T\Sigma\boldsymbol{b}, \tag{8}$$

where $\boldsymbol{b} \equiv \left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1}$.[12] *When producers are fully diversified (they have full ownership of renewable supply), price volatility disappears, i.e., when* $\delta = 1$, $Var(p) = 0$.

This proposition shows that price volatility decreases when the share of producers from renewable supply increases, i.e. $\frac{\partial Var(p)}{\partial \delta} < 0$, and it disappears when $\delta = 1$. This is again due to the fact that when producers have full ownership of all renewable supply (i.e., $\delta = 1$), the total supply (via thermal and renewable sources) of each producer i becomes independent of his type θ_i, vanishing price volatility, i.e., when $\delta = 1$ then $Var(p) = 0$.

The following table summarizes the effect of diversification on the equilibrium price, price volatility and equilibrium forward quantity.

	equilibrium price	price volatility	forward quantity
Renewable outcome ($R \uparrow$)	$-$	$+$	$-$
Diversification ($\delta \uparrow$)	$+$	$-$	$+$
$\delta = 1$	**N**	**N**	**N**

Table 1: The effect of renewable energy (R) on equilibrium price (p^*), price volatility $Var(p^*)$ and equilibrium forward quantity (q^f). With full diversification, all three quantities become independent of R, which we denote by **N** in the table.

We next investigate the impact of correlation structure among parameters θ_i on price volatility. Since correlation among θ_i's captures correlation among renewable energy availability at different plants, our analysis reveals the effect of any kind of dependency, e.g. spatial configuration, of renewable plants on price volatility. We focus on regular configuration in the next section. Price volatility for non-regular structures is studied in Appendix, Section F.

[12] The vector **b** is referred to as the *Bonacich centrality* vector in network models and measures the nodes' centralities in a weighted graph where the weighs in this case are given by the entires of the covariance matrix Σ (see Ballester et al. [2006], Jackson [2008], Acemoglu et al. [2012], Candogan et al. [2012], Bramoullé et al. [2014]).

4.3 Regular Configurations

We next focus on regular configurations, corresponding to a symmetric correlation structure for the renewable plants. This is defined formally through the covariance matrix of $\theta_1, \theta_2, \cdots, \theta_n$ as follows.

Definition 1 (Regular configurations)**.** *Renewable plants have a regular configuration if the covariance matrix Σ is row-(sub)stochastic. That is, $\sum_{j \neq i}^{n} \kappa_{i,j} = K$, where K is fixed and the same for all $i = 1, 2, \cdots, n$.*

Hence, regular configurations represent a correlation structure in which the total covariance of each θ_i with other θ_j's is the same. It follows from Theorem 5 that for regular configurations, the equilibrium is symmetric, i.e., $a_1 = \cdots = a_n$. Thus, the price volatility can be characterized explicitly in terms of K as follows.

Lemma 6. *The price volatility of any regular configuration is given by*

$$Var(p) = n\sigma^2\beta^2 \left(\frac{1-\delta}{2+K}\right)^2 (1+K). \tag{9}$$

$Var(p)$ is decreasing in K, i.e., $\frac{\partial Var(p)}{\partial K} < 0$.

This lemma shows that the price volatility increases when the overall correlation of each plant with its neighbors (i.e. K) decreases. This is intuitive since decreasing the total correlation creates more miscoordination in supplies across producers, increasing volatility in aggregate production. Thus, price volatility rises with decreasing K.

Remark 3. *This monotonicity of price volatility in K depends on the extent of convexity in the cost function. When the cost function is highly convex, price volatility can increase with K. This can be seen by considering the case where the cost function, $C(q_i)$, is sufficiently convex so that several producers rely increasingly on their renewable supply (i.e. $q_i(\theta_i)$ becomes small). As a result, the aggregate production in the economy mostly comes form the aggregate renewable supply. That is*

$$Aggregate\ production = \sum_{i=1}^{n} q_i(\theta) + \sum_{i=1}^{n} R_i \approx \sum_{i=1}^{n} R_i = R + \sum_{i=1}^{n} \theta_i,$$

where R is constant. Hence, greater correlation, i.e. K, increases $Var(\sum_{i=1}^{n} \theta_i) = \mathbf{1}^T \Sigma \mathbf{1} = n\sigma^2(1+K)$, increasing volatility in the aggregate production. Thus, price volatility rises with increasing K (see Theorem 8 in Appendix A.2).

We next present two extreme cases of regular models and consider their implications on the market price volatility.

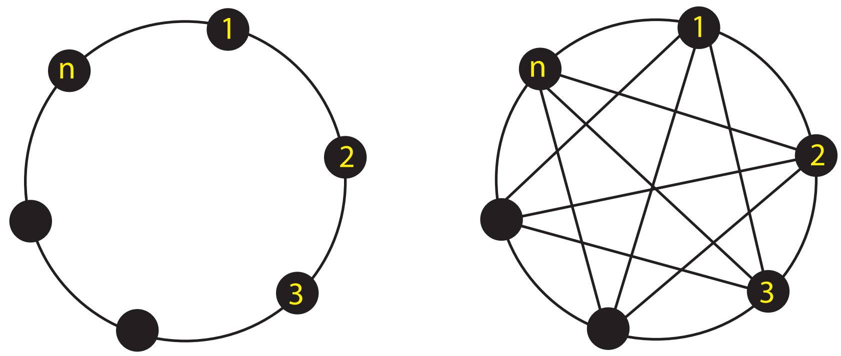

Figure 5: Circle model and complete model.

Ring/Cycle Configuration This corresponds to a structure where renewable energy availability is correlated between two "neighboring" plants and correlation dies down quickly beyond that. We represent this by a circle network along which renewable plants are located. We assume that $\kappa_{i,j} = \zeta^{d(\ell_i,\ell_j)}$ where $0 < \zeta < 1$ is the **decay factor** and $d(\ell_i, \ell_j)$ is the distance (in terms of number of hops along the circle) between l_i and l_j. Hence the total correlation denoted by K is given by

$$K_{\text{cycle}} = \sum_{j \neq i} \kappa_{i,j} = 2\left(\zeta + \zeta^2 + \cdots + \zeta^{\frac{n-1}{2}}\right) = 2\zeta\left(\frac{1-\zeta^{\frac{n-1}{2}}}{1-\zeta}\right),$$

assuming that n is an odd number.

Complete Configuration This corresponds to a structure in which every renewable plant has the same correlation $\zeta \in (0, 1)$ with all others, i.e.,

$$K_{\text{complete}} = \sum_{j \neq i} \kappa_{i,j} = (n-1)\zeta.$$

Comparison between these regular configurations immediately implies that $K_{\text{cycle}} < K_{\text{complete}}$, and that any other regular configurations given n and ζ are in-between these two. Thus, we have the following result directly followed by Lemma 6.

Proposition 2. *Among all geographic configurations with regular structures, the maximum price volatility occurs when renewable plants have a ring structure and the minimum price volatility arises in geographic configurations exhibiting a complete structure.*

As noted above, price volatility is decreasing in the correlations decay factor, ζ. This is because with a greater decay of correlation, there will be greater miscoordination across producers, contributing to price volatility. Since in the complete model each renewable plant is neighbor with *all* the other plants, misscoordination in supplies across competitors is lower than any other regular geographic configurations. The converse holds for the ring structures, where each renewable plant is neighbor with only two plants. Thus, the maximum price volatility occurs when renewable plants have a ring structure and the minimum price volatility arises in geographic configurations exhibiting a complete structure. Price volatility in any other regular configuration lies between these two polar cases.[13]

5 Conclusion

With mounting concerns over climate change caused by fossil fuels, there has been growing reliance on renewable energy. Many countries have responded by not only introducing renewable energy policy targets for the economy at large but imposing these on conventional energy companies. The hope has been to both reduce fossil fuel emissions and also benefit from renewable energy by offering lower prices to consumers through the merit order effect, which refers to the negative impact of renewables, typically supply that low marginal cost, on energy prices.

This paper has studied the implications of diversified energy portfolios on equilibrium prices and the merit order effect in an oligopolistic energy market, and has suggested that the two aforementioned objectives of renewable energy policy may be contradictory. We have shown, in particular, that when thermal generators have a diverse energy portfolio, meaning that they also control some or all of renewable supplies, they offset the price declines due to the merit order effect because they strategically reduce their conventional energy supplies when renewable supply is high. In the limit case where all renewable supply is controlled by thermal power generators this offset is complete — meaning that the merit order effect is totally neutralized and renewable supplies have no impact on market prices. We also established that, through this neutralization of the merit order effect and the resulting higher markups, diversified energy portfolios are welfare-reducing.

These results are first derived in the baseline Cournot model oligopolistic competition. They are then extended to a setup with forward contracts and with incomplete information about imperfectly correlated shocks affecting renewable supplies across the general geographic landscape. In addition to showing the robustness of the partial and full neutralization of the merit order effect, we have used our general framework to study the implications of different calculations on price volatility.

[13] Given the definition of regular structures, for any k-regular configuration with n renewable plants, $K^{(n,2)}_{\text{cycle}} < K^{(n,k)}_{\text{regular}} < K^{(n,n-1)}_{\text{complete}}$.

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Appendix

A Extra results and Extensions

A.1 Welfare (General analysis)

Suppose the demand arises from an aggregate consumer whose gross surplus $U(q) \geq 0$ is concave in q, i.e. $U'' < 0$ (we assume $U(0) = 0$). This gives rise to the inverse demand $P(q) = U'(q)$.[14] The rest of the economy is as in Section 2.1: there are n thermal producers in the market, each thermal producer i faces a (convex and increasing) cost function $C(q_i)$ of supplying q_i unit of energy via thermal sources, the economy has a total R units of renewable energy (at zero marginal cost), and each thermal producer owns a fraction δ/n units of R where $\delta \in [0, 1]$.

The welfare in this economy is the sum of three components: the renewable producers surplus[15] (i.e. $(1-\delta)pR$, where $p \equiv P(\sum_{i=1}^n q_i + R)$), the (total) thermal producers surplus (i.e. $\sum_{i=1}^n \Pi_i = \sum_{i=1}^n [p(q_i + \delta R/n) - C(q_i)]$), and the consumer (net) surplus (i.e. $U(\sum_{i=1}^n q_i + R) - (\sum_{i=1}^n q_i + R)p$). As a result

$$
\begin{aligned}
\mathcal{W} &\equiv \left(\sum_{i=1}^n q_i + \delta R\right) p - \sum_{i=1}^n C(q_i) + (1-\delta)Rp + U\left(\sum_{i=1}^n q_i + R\right) - \left(\sum_{i=1}^n q_i + R\right) p \\
&= U\left(\sum_{i=1}^n q_i + R\right) - \sum_{i=1}^n C(q_i) \qquad (10)
\end{aligned}
$$

Theorem 7. *Let $\mathcal{W}(CE)$ denote the welfare at the corresponding competitive equilibrium and $\mathcal{W}(NE)$ denote the welfare at the corresponding Nash equilibrium. Then, the ratio $\frac{\mathcal{W}(CE)}{\mathcal{W}(NE)}$ is increasing in δ. That is, increasing the diversification ratio leads to an increase in the welfare loss.*

Proof of Theorem 7. The proof follows in three steps as follows.
Step 1 (characterizing $\mathcal{W}(CE)$): Let $q_1^{CE}, \cdots, q_n^{CE}$ be the quantities produced by thermal producers at the competitive equilibrium. Since $\mathcal{W}(CE)\{q_1^{CE}, \cdots, q_n^{CE}\} = \max_{q_1 \geq 0, \cdots, q_n \geq 0} \mathcal{W}$, thus the first order optimality condition of Eq. (10) implies that $q_1^{CE}, \cdots, q_n^{CE}$ should sat-

[14] For example, when $U(q) = \alpha q - \frac{\beta}{2} q^2$, the inverse demand becomes $P(q) = \alpha - \beta q$, the linear inverse demand adopted in the previous sections.

[15] Renewable producers do not have market power. As a result, they sell their production at the level of spot price characterized by the (diversified) thermal producers.

isfy the following equations:

$$U'\left(\sum_{i=1}^{n} q_i^{CE} + R\right) - C'(q_i^{CE}) = 0, \quad \forall i = 1, 2, \cdots, n.$$

By symmetry $q_1^{CE} = \cdots = q_n^{CE} = Q^{CE}/n$ (where $Q^{CE} = \sum_{i=1}^{n} q_i^{CE}$). Therefore, Q^{CE} is characterized from the following equality

$$U'\left(Q^{CE} + R\right) - C'(Q^{CE}/n) = 0. \tag{11}$$

Step 2 (characterizing $\mathcal{W}(NE)$): Let $q_1^{NE}, \cdots, q_n^{NE}$ be the quantities produced via thermal sources when thermal producers are strategic. Thus,

$$q_i^{NE} \in \arg\max_{q_i \geq 0} \quad (q_i + \delta R/n) P\left(q_i + R + \sum_{j \neq i} q_j^{NE}\right) - C(q_i),$$

given $(q_1^{NE}, \cdots, q_{i-1}^{NE}, q_{i+1}^{NE}, \cdots, q_n^{NE})$.

The corresponding first order optimality condition gives

$$P(Q^{NE} + R) + (q_i^{NE} + \delta R/n) P'(Q^{NE} + R) - C'(q_i^{NE}) = 0, \quad i = 1, 2, \cdots, n, \tag{12}$$

where $Q^{NE} \equiv \sum_{i=1}^{n} q_i^{NE}$. Finally, symmetry implies $q_1^{NE} = \cdots = q_n^{NE} = Q^{NE}/n$. Since (by definition) $P(Q^{NE} + R) = U'(Q^{NE} + R)$, thus Eq. (12) gives

$$\begin{aligned} U'(Q^{NE} + R) - C'(Q^{NE}/n) &= -(q_i^{NE} + \delta R/n) P'(Q^{NE} + R) \\ &= -(q_i^{NE} + \delta R/n) U''(Q^{NE} + R) \\ &> 0, \end{aligned}$$

where the last inequality is true because $U'' < 0$.

Step 3 (Effect of δ on $\mathcal{W}(CE)$ and $\mathcal{W}(NE)$): In this step we show $\mathcal{W}(CE)$ does not depend on δ, however, $\mathcal{W}(NE)$ is decreasing in δ. Equation Eq. (10) implies

$$\begin{aligned} \frac{\partial \mathcal{W}(T)}{\partial \delta} &= \sum_{i=1}^{n} \frac{\partial q_i^T}{\partial \delta} \left[U'\left(\sum_{i=1}^{n} q_i^T + R\right) - C'(q_i^T)\right] \\ &= \frac{\partial Q^T}{\partial \delta} \left(U'(Q^T + R) - C'(Q^T/n)\right) \qquad \text{for } T \in \{CE, NE\}. \end{aligned}$$

Therefore, as shown in Step 2, $U'\left(Q^{CE} + R\right) - C'(Q^{CE}/n) = 0$, thus $\frac{\partial \mathcal{W}(CE)}{\partial \delta} = 0$, i.e.

$\mathcal{W}(CE)$ does not depend on δ. However, as shown in Step 3, $U'\left(Q^{NE}+R\right)-C'(Q^{NE}/n)>0$, thus $\text{sign}\{\frac{\partial\mathcal{W}(NE)}{\partial\delta}\}=\text{sign}\{\frac{\partial Q^{NE}}{\partial\delta}\}$. Moreover, we show in Theorem 1 that $\frac{\partial Q^{NE}}{\partial\delta}<0$ (see equation Eq. (24)), therefore $\frac{\partial\mathcal{W}(NE)}{\partial\delta}<0$, i.e. $\mathcal{W}(NE)$ is decreasing in δ. As a result, $\frac{\partial}{\partial\delta}\left(\frac{\mathcal{W}(CE)}{\mathcal{W}(NE)}\right)>0$, completing the proof. ∎

A.2 Price Volatility: Linear vs. Quadratic costs

We focus on regular configurations, which represents a symmetric correlation structure for the renewable plants. This is defined formally through the covariance matrix of θ_i's as follows.

Definition 2 (Regular configurations)**.** *Renewable plants have a regular configuration if the covariance matrix Σ is row-(sub)stochastic. That is, $\sum_{j\neq i}^{n}\kappa_{i,j}=K$, where K is fixed and the same for all $i=1,2,\cdots,n$.*

Hence, regular configurations represent a correlation structure in which the total covariance of each θ_i with other θ_j's is the same. It follows from Theorem 5 that for regular configurations, the equilibrium is symmetric, i.e., $a_1=\cdots=a_n$. Moreover the price volatility can be characterized explicitly in terms of K as follows.

Theorem 8. *Let the production cost via thermal sources be given by $C(q_i)=\gamma q_i+\frac{\lambda}{2}q_i^2$. Then, the price volatility of any regular configuration is given by*

$$Var(p)=n\sigma^2\beta^2\left(\frac{\beta(1-\delta)+\lambda}{\beta(2+K)+\lambda}\right)^2(1+K). \tag{13}$$

Moreover:

(i) When producers have strictly convex costs (i.e. $\lambda>0$) and are fully diversified (they have full ownership of renewable supply), price volatility does not disappear, i.e., if $\lambda>0$ and $\delta=1$, then $Var(p)\neq 0$. This result holds for any configuration (i.e. there is no need to have a regular configuration).

(ii) When cost is linear (i.e. $\lambda=0$), price volatility is monotonically decreasing in K, i.e., $\frac{\partial Var(p)}{\partial K}<0$.

(iii) Let β be fixed. When cost is strictly convex (i.e. $\lambda>0$), depending on the degree of convexity in the cost function price volatility can be either increasing or decreasing in K. To be precise

$$\text{sign}\{\frac{\partial Var(p)}{\partial K}\}=\begin{cases}+ & \text{if } \frac{\lambda}{\beta}>K;\\ - & \text{if } \frac{\lambda}{\beta}<K.\end{cases}$$

This result has two important consequences. First, when cost function is sufficiently convex, i.e. $\lambda > 0$, in contrast to the linear cost (see Proposition 1), price volatility does *not* disappear when $\delta = 1$. This is simply because when thermal producers are fully diversified, i.e. $\delta = 1$, and their cost function is convex, i.e. $\lambda > 0$, then the total supply (via thermal and renewable sources) of each producer i still depends on θ_i (this will be more clear by the following Example). Second, the monotonicity of price volatility in K depends on the extent of convexity in the cost function. That is, assuming β is fixed, depending on the extent of convexity in the cost function, price volatility can be either increasing or decreasing in K. In fact, in contrast to the linear cost function, with increasing the extent of convexity in the cost function, price volatility can become increasing in K. To see this, suppose the cost function from thermal sources, i.e. $C(q_i)$, is sufficiently convex in q_i, so that production from thermal sources is so expensive. Therefore, each diversified thermal producer cuts its production via thermal sources (i.e. $q_i(\theta_i)$ becomes small). As a result, the aggregate production in the economy mostly comes form the aggregate renewable supply. That is, Aggregate production $= \sum_{i=1}^n q_i(\theta) + \sum_{i=1}^n R_i \approx \sum_{i=1}^n R_i = R + \sum_{i=1}^n \theta_i$, where R is constant. Hence, increasing correlation, i.e. K, increases $Var(\sum_{i=1}^n \theta_i) = \mathbf{1}^T \Sigma \mathbf{1} = n\sigma^2(1+K)$, increasing volatility in the aggregate production. Thus, price volatility can increase with increasing K, given λ is sufficiently large.

Proof of Theorem 8. The proof follows similar steps as in the proofs of Theorem 5 and Proposition 1. In the first part the analysis is for a general configuration. Next we focus on the regular configurations.

General configuration Given that $C(q_i) = \gamma q_i + \frac{\lambda}{2} q_i^2$, producer i's objective is to choose q_i maximizing

$$E_{\theta_{-i}}(\Pi_i | R_i) = E\{p(q_i - q_i^f + \delta_i R_i) + p_i^f q_i^f - \gamma q_i - \frac{\lambda}{2} q_i^2 | R_i\}$$

where $q_j(\theta_j) = b_j - a_j \theta_j$, for all $j \neq i$, and $p = \alpha - \beta(q_i + R_i + \sum_{j \neq i} R_j + \sum_{j \neq i} q_j)$. Since $R_i = R/n + \theta_i$, for all $i = 1, 2, \cdots, n$, thus, the first order optimality condition (FOC) implies

$$\alpha - \gamma - \beta \left(\sum_{j \neq i} E[q_j(\theta_j) | R_i] + \sum_{j \neq i} E[\theta_j | R_i] + \theta_i + R \right) - \beta(-q_i^f + \delta R/n + \delta \theta_i) = (2\beta + \lambda) q_i \tag{14}$$

Using the projection theorem: $E[q_j(\theta_j) | R_i] = b_j - a_j \kappa_{i,j} \theta_i$ and $E[\theta_j | R_i] = \kappa_{i,j} \theta_i$.

As a result rearranging terms in Eq. (14) gives

$$\left(\alpha - \gamma - \beta\left(\sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right)\right) - \theta_i\beta\left((1+\delta) + \sum_{j\neq i}\kappa_{i,j} - \sum_{j\neq i} a_j\kappa_{i,j}\right)$$
$$= ((2\beta+\lambda)b_i) - \theta_i((2\beta+\lambda)a_i) \tag{15}$$

To analyze price volatility we only need to find a_i for $i = 1, 2, \cdots n$. Thus, we only need to equate the coefficient of θ_i in the LHS and RHS of Eq. (15), that implies (note that $\beta > 0$)

$$\sum_{j\neq i}\kappa_{i,j}a_j + \left(2 + \frac{\lambda}{\beta}\right)a_i = (1+\delta) + \sum_{j\neq i}\kappa_{i,j} \equiv v_i \tag{16}$$
$$\Rightarrow \quad \tilde{A}\boldsymbol{a} = \boldsymbol{v},$$

where $\tilde{A} \equiv \frac{1}{\sigma^2}\Sigma + \left(1 + \frac{\lambda}{\beta}\right)\mathbf{I}$, and $\mathbf{I}$ denotes the identity matrix. Since $\tilde{A}$ is positive define, it is invertible and thus

$$\begin{aligned}
\boldsymbol{a} &= \tilde{A}^{-1}\boldsymbol{v} \\
&= \left(\frac{1}{\sigma^2}\Sigma + (1+\frac{\lambda}{\beta})\mathbf{I}\right)^{-1}(\delta\mathbf{1} + \frac{1}{\sigma^2}\Sigma\mathbf{1}) \\
&= \left(\frac{1}{\sigma^2}\Sigma + (1+\frac{\lambda}{\beta})\mathbf{I}\right)^{-1}\left((\delta - (1+\frac{\lambda}{\beta}))\,\mathbf{I} + \frac{1}{\sigma^2}\Sigma + (1+\frac{\lambda}{\beta})\,\mathbf{I}\right)\mathbf{1} \\
&= \mathbf{1} + (\delta - \left(1+\frac{\lambda}{\beta}\right))\left(\left(1+\frac{\lambda}{\beta}\right)\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1}.
\end{aligned} \tag{17}$$

As shown in the proof of Proposition 1, $Var(p) = (\mathbf{a} - \mathbf{1})^T\Sigma(\mathbf{a} - \mathbf{1})$, thus

$$Var(p) = (\delta - \left(1+\frac{\lambda}{\beta}\right))^2 \quad \mathbf{1}^T\left(\left(1+\frac{\lambda}{\beta}\right)\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\Sigma\left(\left(1+\frac{\lambda}{\beta}\right)\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1}$$

Thus, when $\lambda > 0$ and $\delta = 1$ (in contrast to the linear cost), $Var(p) \neq 0$.

Regular configuration For regular configurations $a_1 = \cdots = a_n \equiv \tilde{a}$. Thus Eq. (16) implies $\tilde{a} = \frac{\beta(1+\delta+K)}{\beta(K+2)+\lambda}$.

Moreover, as shown in the proof of Proposition 1, $Var(p) = \beta^2 Var\left(\sum_{i=1}^n (a_i - 1)\theta_i\right)$, thus for regular configurations we have

$$Var(p) = \beta^2(\tilde{a}-1)^2\mathbf{1}^T\mathbf{\Sigma}\mathbf{1} = n\sigma^2\beta^2\left(\frac{\beta(1-\delta)+\lambda}{\beta(2+K)+\lambda}\right)^2(1+K).$$

The explicit characterization of $Var(p)$ implies that

$$\frac{\partial Var(p)}{\partial K} = \left(n\sigma^2\beta^2 \frac{(\beta(1-\delta)+\lambda)^2}{(\beta(2+K)+\lambda)^3} \right) [\lambda - \beta K]$$

completing the proof. ∎

Example (Duopoly with incomplete information and quadratic cost) Let us assume each producer $i \in \{1,2\}$ owns a generator that produces q_i units of thermal energy at cost $C(q_i) = \frac{\lambda}{2}q_i^2$ (where $\lambda > 0$). In this economy thermal producers are also capable to generate energy from renewable plants. To this end, we assume there are two intermittent plants. Let ℓ_1 and ℓ_2 denote the locations of these plants. Each producer i privately observes the available renewable energy R_i at local plant l_i. We assume $R_i = R/2 + \theta_i$, where R is a constant, and θ_i is normally distributed with mean zero and variance σ^2, i.e., $\theta_i \sim \mathcal{N}(0, \sigma^2)$. The vector $\theta = (\theta_1, \theta_2)$ is assumed to be jointly normal and $\text{cov}(\theta_1, \theta_2) = \kappa\sigma^2$, where $\kappa \in [0,1]$. The scalar κ captures the correlation between available renewable energy at plants l_i and l_j.[16]

For ease of exposition we assume $p \equiv \alpha - (q_1 + q_2 + R_1 + R_2)$, consequently,[17] producer i's (ex-post) payoff becomes:

$$\Pi_i = p(q_i + \delta R_i) - \lambda\frac{q_i^2}{2} = (\alpha - q_1 - q_2 - R_1 - R_2)(q_i + \delta R_i) - \lambda\frac{q_i^2}{2}.$$

Solving this case implies

$$q_i(\theta_i) = \frac{\alpha - R - \delta R/2}{\lambda + 3} - \left(\frac{1+\delta+\kappa}{\lambda+2+\kappa}\right)\theta_i$$

$$Var[p] = 2\sigma^2\left(\frac{\lambda+1-\delta}{\lambda+2+\kappa}\right)^2(1+\kappa) \tag{18}$$

Therefore, with increasing λ, intuitively, production from thermal sources decreases, i.e. $\frac{\partial E[q_i(\theta_i)]}{\partial \lambda} < 0$.

Furthermore, using Theorem 8, we have

$$\frac{\partial Var(p)}{\partial \kappa} = \left(\frac{2\sigma^2(\lambda+1-\delta)^2}{(\lambda+2+\kappa)^3}\right)(\lambda - \kappa).$$

Therefore, depending on the extent of convexity in the cost function, price volatility can

[16] It is worth noting that adding forward contract does not have any effect on the price volatility.
[17] In this simple example we assume $\beta = 1$.

be decreasing or increasing with respect κ. To be precise:

$$\text{sign}(\frac{\partial Var(p)}{\partial \kappa}) = \text{sign}(\lambda - \kappa)$$

where as for the linear cost price volatility monotonically decreases in κ (see Lemma 6).

A.3 Degrees of convexity and concavity

This section analyzes the effects of convex cost and concave inverse demand functions on the market price, when thermal producers have a diverse energy portfolio. We show, for a given concave inverse demand function, with increasing extent of convexity in the cost function market price goes up. However, for a given convex cost function, with increasing extent of concavity in the inverse demand market price goes down. To this end, we consider two cases as follows. Without loss of generality we assume $n = 2$.

Concavity analysis of the inverse demand Let the cost function be a given convex function (i.e. $C'' \geq 0$), and inverse demand be $p = P(Q) = \alpha - \beta Q^2$, where $\beta > 0$. Thus, the more β is, the more concave the inverse demand $P(Q)$ is. The objective is to show $\frac{\partial p}{\partial \beta} < 0$.

The profit of each (diverse) thermal producer is given by

$$\Pi_i = (q_i + \delta R/2)P(Q+R) - C(q_i) = (q_i + \delta R/2)(\alpha - \beta(Q+R)^2) - C(q_i)$$

FOC then gives $\frac{\partial \Pi_i}{\partial q_i} = (\alpha - \beta(Q+R)^2) + (q_i + \delta R/2)(-2\beta(Q+R)) - C'(q_i) = 0$. Due to symmetry (at equilibrium) $q_1 = q_2$. Thus,

$$0 = (\alpha - \beta(Q+R)^2) + (Q + \delta R)(-\beta(Q+R)) - C'(Q/2).$$

Taking a derivative with respect β implies

$$\Big[(Q+R)^2 + (Q+\delta R)(Q+R)\Big] + \frac{\partial Q}{\partial \beta}\Big[3\beta(Q+R) + \beta(Q+\delta R) + 1/2C''(Q/2)\Big] = 0.$$

Thus (note that $C'' \geq 0$),

$$\frac{\partial Q}{\partial \beta} = -\frac{(Q+R)^2 + (Q+\delta R)(Q+R)}{3\beta(Q+R) + \beta(Q+\delta R) + 1/2C''(Q/2)} < 0.$$

What is the effect of inverse demand concavity (controlled by β) on the market price p?

Since $p = \alpha - \beta(Q+R)^2$, thus

$$\begin{aligned}\frac{\partial p}{\partial \beta} &= -(Q+R)^2 - 2\beta(Q+R)\frac{\partial Q}{\partial \beta} \\ &= -(Q+R)^2\left[1 - 2\beta\frac{Q+R+Q+\delta R}{3\beta(Q+R)+\beta(Q+\delta R)+1/2C''(Q/2)}\right] \\ &\leq 0,\end{aligned}$$

where the last inequality is correct because [...] ≥ 0 (note that $\delta \in [0,\ 1]$ and $C'' \geq 0$). Therefore, with increasing extent of concavity in the inverse demand market price decreases.

Convexity analysis of the cost function Let the inverse demand be concave (and downward) (i.e. $P'' \leq 0$ and $P' < 0$) and the cost function be $C(q_i) = \gamma q_i + \frac{\lambda}{2}q_i^2$, where $\gamma \geq 0$ and $\lambda \geq 0$. Thus, the more is λ, the more convex is the cost function $C(q_i)$. The objective is to show $\frac{\partial p}{\partial \lambda} > 0$.

The analysis is inline with the previous case. The profit of each (diverse) thermal producer is updated by $\Pi_i = (q_i + \delta R/2)P(Q+R) - C(q_i) = (q_i + \delta R/2)P(Q+R) - \left(\gamma q_i + \frac{\lambda}{2}q_i^2\right)$. The FOC then gives $\frac{\partial \Pi_i}{\partial q_i} = P(Q+R) + (q_i + \delta R/2)P'(Q+R) - (\gamma + \lambda q_i)$. Due to the symmetry (at equilibrium) $q_1 = q_2$. Thus,

$$0 = P(Q+R) + (\frac{1}{2})(Q+\delta R)P'(Q+R) - \left(\gamma + \lambda\frac{Q}{2}\right).$$

Taking a derivative with respect λ and rearranging terms imply

$$\frac{\partial Q}{\partial \lambda} = \frac{Q}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - \lambda} < 0,$$

where the last inequality is because the inverse demand in downward and concave (i.e. $P' < 0$ and $P'' \leq 0$). Thus, with increasing convexity in the cost function aggregate production decreases. As a result, since P is decreasing in Q (i.e. $P' < 0$), thus the market price increases in λ, completing the proof.

B Proofs omitted from main text

B.1 Merit order effect vs. Diversification

Proof of Theorem 1. We present the proof for the duopoly case, extension to $n \geq 2$ is straightforward. With the concave (downward) inverse demand P and the convex cost C, profit of each (diverse) thermal producer is given by

$$\Pi_i = (q_i + \delta R/2)P(Q+R) - C(q_i), \tag{19}$$

where each conventional generator owns $\delta R/2$ units of renewable supply, $\delta \in [0,\ 1]$.

We first note that

$$\frac{\partial p}{\partial R} = \left(\frac{\partial Q}{\partial R} + 1\right) P'(Q+R), \tag{20}$$

where $p \equiv P(Q+R)$. Since P' is downward (i.e. $P' < 0$), thus to show $\frac{\partial p}{\partial R} \leq 0$, we next prove that $\frac{\partial Q}{\partial R} + 1 \geq 0$.

FOC implies

$$0 = \frac{\partial \Pi_i}{\partial q_i} = P(Q+R) + (q_i + \delta R/2)P'(Q+R) - C'(q_i)$$

By symmetry $q_1 = q_2$, thus the above equation is equivalent to

$$0 = \frac{\partial \Pi_i}{\partial q_i} = P(Q+R) + (\frac{1}{2})(Q+\delta R)P'(Q+R) - C'(Q/2) \tag{21}$$

Taking derivative from Eq. (21) with respect to R implies

$$\begin{aligned} 0 = & \left(1 + \frac{\partial Q}{\partial R}\right) P'(Q+R) + (\frac{1}{2}) \left(1 + \frac{\partial Q}{\partial R}\right) (Q+\delta R)P''(Q+R) \\ & + (\frac{1}{2}) \left(\delta + \frac{\partial Q}{\partial R}\right) P'(Q+R) - (\frac{1}{2}) \left(\frac{\partial Q}{\partial R}\right) C'' \left(\frac{Q}{2}\right) \end{aligned}$$

Rearranging terms yields

$$\begin{aligned} 0 = & \left[3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)\right] \frac{\partial Q}{\partial R} \\ & + \left[(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)\right] \end{aligned}$$

Consequently,

$$\frac{\partial Q}{\partial R} = -\frac{(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)} \tag{22}$$

Recall that the cost function is convex, thus $C'' \geq 0$. As a result, with linear cost, $C'' = 0$, since $P' < 0$ and $P'' < 0$, thus Eq. (22) implies

$$-1 \leq \frac{\partial Q}{\partial R} < 0 \quad \Rightarrow \quad 1 + \frac{\partial Q}{\partial R} \geq 0, \quad \text{and with } \delta = 1, \quad -1 = \frac{\partial Q}{\partial R}. \tag{23}$$

Thus, using Eq. (20), we obtain $\frac{\partial p}{\partial R} \leq 0$. Moreover, when all renewable supply generates profits for only conventional power generators (i.e. $\delta = 1$), $\frac{\partial p}{\partial R} = 0$, neutralizing the MoE. However, with strictly convex cost (i.e. $C'' > 0$), for $\delta \in [0,\ 1]$:

$$\begin{aligned} 0 > \frac{\partial Q}{\partial R} &= -\frac{(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)} \\ &> -\frac{(2+\delta)P'(Q+R) + (Q+\delta R)P''(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R)} \\ &\geq -1. \end{aligned}$$

As a result, $\frac{\partial Q}{\partial R} + 1 > 0$, consequently, using Eq. (20), $\frac{\partial p}{\partial R} < 0$ for all $\delta \in [0,\ 1]$. Therefore, full neutralization may not be obtained with strictly convex cost functions.

To wrap up the proof we next show $\frac{\partial p}{\partial \delta} > 0$, diversification amplifies the prices. Since

$$\frac{\partial p}{\partial \delta} = \left(\frac{\partial Q}{\partial \delta}\right) \underbrace{P'(Q+R)}_{<0}$$

to prove the claim is then sufficient to show $\frac{\partial Q}{\partial \delta} < 0$. Taking a derivative from Eq. (21) with respect to δ implies

$$\begin{aligned} 0 = \left(\frac{\partial Q}{\partial \delta}\right) P'(Q+R) + (\frac{1}{2}) \left(\frac{\partial Q}{\partial \delta}\right) (Q+\delta R)P''(Q+R) + (\frac{1}{2}) \left(R + \frac{\partial Q}{\partial \delta}\right) P'(Q+R) \\ - (\frac{1}{2}) \left(\frac{\partial Q}{\partial \delta}\right) C''\left(\frac{Q}{2}\right) \end{aligned}$$

Rearranging terms gives

$$0 = \left[3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)\right]\frac{\partial Q}{\partial \delta} + RP'(Q+R)$$

Therefore,

$$\frac{\partial Q}{\partial \delta} = -\frac{RP'(Q+R)}{3P'(Q+R) + (Q+\delta R)P''(Q+R) - C''\left(\frac{Q}{2}\right)} < 0, \tag{24}$$

completing the proof of the first part.

To prove the second part we note that since $P' \neq 0$, thus $\frac{\partial p}{\partial R} = \left(\frac{\partial Q}{\partial R} + 1\right)P'(Q+R) = 0$ if and only if $\frac{\partial Q}{\partial R} = -1$. Therefore, when $\delta \to 1$, using Eq. (22), we obtain $\frac{\partial Q}{\partial R} = -1$ if and only if $C'' = 0$. Thus, under any condition ensuring a unique interior equilibrium, neutralization result prevails when (i) thermal producers are diversified, (ii) cost of production (via thermal sources) is either linear or constant, i.e. $C'' = 0$.

It is worthy to note that, inspired by the standard Cournot model (see Ch 4 of Vives [1999] and Kolstad and Mathiesen [1987]), a unique equilibrium is ensured in our model if: (i) $C'' - P'(Q+R) > 0$, (ii) $\frac{P'(Q+R)+(q_i+\delta\frac{R}{2})P''(Q+R)}{C''-P'(Q+R)} < \frac{1}{n}$, where n denotes the number of thermal generators.

∎

C Derivations of the reduced-from models

Proof of Theorem 2. Since $\Pi_i = (\alpha - \beta(\sum_i q_i + R))(q_i + \delta R/n) - \gamma q_i$, thus FOC implies $\alpha - \beta(q_i + \sum_{j\neq i} q_j + R) - \beta(q_i + \delta R/n) - \gamma = 0$, for all $i = 1, 2, \cdots, n$. Taking a sum over all i implies $n(\alpha-\gamma) - n\beta(Q+R) - \beta(Q+\delta R) = 0$. Hence, at the equilibrium, $Q = \frac{n(\alpha-\gamma)-\beta(\delta R+nR)}{\beta(n+1)}$. By symmetry, $q_i = Q/n = \frac{\alpha-\gamma-\beta(\delta R/n+R)}{\beta(n+1)}$, for all $i = 1, 2, \cdots, n$. Further, plugging Q into $p = \alpha - \beta(Q+R)$ implies $p = \frac{\alpha+\beta(-R+\delta R)+n\gamma}{n+1}$, as desired. ∎

Proof of Theorem 4. To have a better understanding of the proof steps we first consider the duopoly case. The oligopoly case is more involved but it follows similar steps.

Consider the duopoly case, i.e. $n = 2$. By adding forward contract to the previous case the profit of each generator becomes $\Pi_i(q_1, q_2) = (\alpha - \beta(q_1 + q_2 + R))(q_i - q_i^f + \delta R/2) + q_i^f p_i^f - \gamma q_i$. In this case, the economy has two dates, $t = 1, 2$: generators sign forward contract at $t = 1$ and the market clearing price $p = \alpha - \beta(\sum_{i=1}^n q_i + R)$ is characterized at the final date $t = 2$. The solution strategy is to work backward. Thus, given $(q_1^f, p_1^f, q_2^f, p_2^f)$, FOC implies

$$0 = \frac{\partial \Pi_i}{\partial q_i} = \alpha - \gamma - \beta(q_1 + q_2 + R) - \beta(q_i - q_i^f + \delta R/2). \tag{25}$$

Summing over $i \in \{1, 2\}$ and rearranging terms imply $Q = \frac{1}{3\beta}\left(2\alpha - 2\beta R - \beta(-Q^f + \delta R) - 2\gamma\right)$. Plugging Q into Eq. (25) and some alegra yield

$$q_i = \frac{1}{3\beta}\left(\alpha - \gamma - \beta(Q^f - 3q_i^f + R + \delta R/2)\right).$$

Now, given the optimal supply at the final date, we next characterize the optimal forward contract for each generator. Note that assuming no possibility for arbitrage implies at $t = 1$ the forward quantity q_i^f is signed at the market price, i.e. $p_i^f = p$. Thus, optimal q_i^f maximizes $p(q_i + \delta R/2) - \gamma q_i$, where $p = \alpha - \beta(q_1 + q_2 + R)$. Since $\frac{\partial p}{\partial q_i^f} = -\beta/3$ and $\frac{\partial q_i}{\partial q_i^f} = 2/3$, thus FOC gives $\frac{\partial p}{\partial q_i^f}(q_i + \delta R/2) + p\frac{\partial q_i}{\partial q_i^f} - \gamma\frac{\partial q_i}{\partial q_i^f} = -\frac{\beta}{3}(q_i + \delta R/2) + (p - \gamma)\frac{2}{3} = 0$. Simplifying this equation after plugging Eq. (25) into it, implies

$$q_1^f = q_2^f = \frac{1}{5\beta}\left(\alpha - \gamma + \beta(-R + \delta R)\right).$$

Plugging q_i^f into Eq. (25) yields $q_i = \frac{2}{5\beta}\left(\alpha - \gamma + \beta(-R - \delta R/4)\right)$. Finally, market price becomes $p = \alpha - \beta(q_1 + q_2 + R) = \frac{1}{5}\left(\alpha + 4\gamma + \beta(-R + \delta R)\right)$.

We next consider the oligopoly case, i.e. $n \geq 2$. Given the profit of producer i, i.e. $\Pi_i(q_i, q_{-i}) = (\alpha - \beta(q_i + \sum_{j \neq i} q_j + R))(q_i - q_i^f + \delta R/n) + q_i^f p_i^f - \gamma q_i$, employing the FOC implies $\alpha - \gamma - \beta\left(\sum_{j \neq i} q_j + R - q_i^f + \delta R/n\right) = 2\beta q_i$. Thus, rearranging terms yields

$$\alpha - \gamma - \beta\left(2q_i + \sum_{j \neq i} q_j + R - q_i^f + \delta R/n\right) = 0. \tag{26}$$

Let $Q \equiv \sum_{j=1}^n Q_j$. Taking a sum over all i from Eq. (26) implies

$$Q = \frac{n(\alpha - \gamma - \beta R) - \beta(-Q^f + \delta R)}{(n+1)\beta}, \tag{27}$$

where $Q^f \equiv \sum_{i=1}^n q_i^f$.

Next, from Eq. (26) we obtain

$$\alpha - \beta(Q + R) - \beta(-q_i^f + \delta R/n) - \gamma = \beta q_i.$$

The LHS of the above equation can be simplified as follows:

$$\begin{aligned}
\text{LHS} &= (\alpha - \gamma) - \beta R - \beta(q_i^f + \delta R/n) - \beta Q \\
&= \frac{1}{n+1}\Big((n+1)[(\alpha-\gamma) - \beta R - \beta(-q_i^f + \delta R/n)] - n(\alpha-\gamma) + n\beta R + \beta(-Q^f + \delta R)\Big) \\
&= \frac{1}{n+1}\Big((\alpha-\gamma) - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\Big)
\end{aligned}$$

Therefore, we have

$$q_i = \frac{1}{(n+1)\beta}\Big(\alpha - \gamma - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\Big). \tag{28}$$

Next, we move to the contracting stage.

Contracting stage Equipped with the results from the production stage, we next find optimal forward quantities, i.e. $q_1^f, q_2^f, \cdots, q_n^f$. Importantly, due to the no arbitrage assumption $p_i^f = p$. Thus, producer i's optimal choice for q_i^f should maximize

$$\Big(\alpha - \beta(Q(q_i^f, q_{-i}^f) + R)\Big)\left(q_i(q_i^f, q_{-i}^f) + \delta R/n\right) - \gamma q_i(q_i^f, q_{-i}^f).$$

Thus, the FOC gives

$$(\alpha - \beta(Q(q_i^f, q_{-i}^f) + R))\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} - \beta\frac{\partial Q(q_i^f, q_{-i}^f)}{\partial q_i^f}\Big(q_i(q_i^f, q_{-i}^f) + \delta R/n\Big) - \gamma\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = 0 \tag{29}$$

Since $\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{n}{n+1}$ (see Eq. (28)) and $\frac{\partial Q(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{1}{n+1}$ (see Eq. (27)), thus Eq. (29) yeilds

$$(\alpha - \beta(Q(q_i^f, q_{-i}^f) + R))\frac{n}{n+1} - \gamma\frac{n}{n+1} - (q_i(q_i^f, q_{-i}^f) + \delta R/n)\frac{\beta}{n+1} = 0 \tag{30}$$

multiplying in $(n+1)$ and rearranging terms give $-\beta(q_i(q_i^f, q_{-i}^f) + nQ(q_i^f, q_{-i}^f)) + n(\alpha - \gamma) - \beta(\delta R/n + nR) = 0$. By symmetry $q_1(q_1^f, q_{-1}^f) = q_2(q_2^f, q_{-2}^f) = \cdots = q_n(q_n^f, q_{-n}^f)$, thus

$$-\beta(n^2+1)q_i(q_i^f, q_{-i}^f) + n(\alpha - \gamma) - \beta(\delta R/n + nR) = 0. \tag{31}$$

Moreover, Eq. (28) gives that $-\beta(n^2+1)q_i(q_i^f, q_{-i}^f) = -\frac{n^2+1}{n+1}\left(\alpha - \gamma - \beta\left[-q_i^f + R + \delta R/n\right]\right)$, note that by symmetry $q_1^f = q_2^f = \cdots = q_n^f$.

Plugging this into Eq. (31) gives

$$-(n^2+1)\left[\alpha - \gamma - \beta(-q_i^f + R + \delta R/n)\right] + (n^2+n)(\alpha-\gamma) - (n+1)\beta(\delta R/n + nR) = 0. \tag{32}$$

Rearranging terms implies

$$\begin{aligned}(n^2+1)\beta q_i^f &= (n-1)(\alpha-\gamma) + (n^2+1)\beta(R+\delta R/n) - (n+1)\beta(\delta R/n + nR)\\ &= (n-1)[\alpha - \gamma + \beta(-R+\delta R)].\end{aligned}$$

Thus

$$q_i^f = \frac{n-1}{(n^2+1)\beta}\Big(\alpha - \gamma + \beta(-R+\delta R)\Big). \tag{33}$$

Thus, we finally find q_i by plugging q_i^f into Eq. (28). That is

$$\begin{aligned}q_i &= \frac{1}{(n+1)\beta}\Big(\alpha - \gamma - \beta(-q_i^f + R + \delta R/n)\Big)\\ &= \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta\left[-\frac{n-1}{(n^2+1)\beta}\Big(\alpha - \gamma + \beta(-R+\delta R)\Big) + R + \delta R/n\right]\right)\\ &= \frac{n}{(n^2+1)\beta}\left(\alpha + \beta\left(-R - \frac{\delta R}{n^2}\right) - \gamma\right).\end{aligned}$$

With the characterization of q_i, $i = 1, 2\cdots, n$, the proof is complete.

∎

D Correlated shocks and incomplete information with endogenous forward contract

D.1 Equilibrium characterization

Proof of Theorem 5. It is useful to construct the equilibrium sequentially and work backward. We start by deriving the optimal behavior at the production stage, for any given set of forward contract level $(q_i^f, p_i^f)_{i=1,\cdots,n}$ that summarizes the behavior at the production stage. Next, we analyze the optimal behavior at the contracting stage.

Production stage For a given profile of contracting profile $((q_1^f, p_1^f), \cdots, (q_n^f, p_n^f))$, the production stage is a game with imperfect competition in the class of normal linear-quadratic games with correlated types, once a generator information R_i is established as the type. A strategy q_i is a mapping from the information set into the real space: $q_i : \mathbb{R} \to \mathbb{R}_+$ for a given individual contracting choice (q_i^f, p_i^f). We focus on linear equilibria at the production stage throughout, that is for each generator $i = 1, \cdots, n$, there exists a_i and b_i such that $q_i(R_i) = b_i - a_i\theta_i$. Optimality for generator i at the production stage requires that strategy q_i maximizes the conditional expected utility, taking all other generators strategies q_{-i} as given. Furthermore, applying the projection theorem,[18] with the linear information structure conditional expectations $E(\theta_j|\theta_i)$ are linear in θ_i, for all i, j.

Producer i chooses q_i to maximize

$$E_{\theta_{-i}}(\Pi_i|R_i) = E\{p(q_i - q_i^f + \delta_i R_i) + p_i^f q_i^f - \gamma q_i|R_i\}$$

where $q_j(\theta_j) = b_j - a_j\theta_j$, for all $j \neq i$, and $p = \alpha - \beta(q_i + R_i + \sum_{j\neq i} R_j + \sum_{j\neq i} q_j)$. Recall that $R_i = R/n + \theta_i$, for all $i = 1, 2, \cdots, n$, thus, the first order conditions (FOC) gives

$$\alpha - \gamma - \beta\left(\sum_{j\neq i} E[q_j(\theta_j)|R_i] + \sum_{j\neq i} E[\theta_j|R_i] + \theta_i + R\right) - \beta(-q_i^f + \delta R/n + \delta\theta_i) = 2\beta q_i \quad (34)$$

Using the projection theorem:

$$\begin{aligned} E[q_j(\theta_j)|R_i] &= E[q_j(\theta_j)|\theta_i] = b_j - a_j\, Cov(\theta_i, \theta_j)\, \sigma^{-2}\, \theta_i = b_j - a_j\kappa_{i,j}\theta_i \\ E[\theta_j|R_i] &= E[\theta_j|\theta_i] = Cov(\theta_i, \theta_j)\, \sigma^{-2}\, \theta_i = \kappa_{i,j}\theta_i. \end{aligned} \quad (35)$$

Plugging Eq. (35) into Eq. (34) and rearranging terms gives

$$\begin{aligned} &\left(\alpha - \gamma - \beta\left(\sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right)\right) - \theta_i\beta\left((1+\delta) + \sum_{j\neq i}\kappa_{i,j} - \sum_{j\neq i} a_j\kappa_{i,j}\right) \\ &\quad = (2\beta b_i) - \theta_i(2\beta a_i) \end{aligned} \quad (36)$$

[18]Let θ and ν be random variables such that $(\theta, \nu) \sim \mathcal{N}(\mu, \Sigma)$ such that:

$$\mu \equiv \begin{pmatrix} \mu_\theta \\ \mu_\nu \end{pmatrix} \qquad \Sigma \equiv \begin{pmatrix} \Sigma_{\theta,\theta} & \Sigma_{\theta,\nu} \\ \Sigma_{\nu,\theta} & \Sigma_{\nu,\nu} \end{pmatrix}$$

are expectations and variance-covariance matrix, then the conditional density of θ given ν is normal with conditional mean $\mu_\theta + \Sigma_{\theta,\nu}\Sigma_{\nu,\nu}^{-1}(\nu - \mu_\nu)$ and variance-covariance matrix $\Sigma_{\theta,\theta} - \Sigma_{\theta,\nu}\Sigma_{\nu,\nu}^{-1}\Sigma_{\nu,\theta}$, provided that $\Sigma_{\nu,\nu}$ is non-singular, DeGroot [1970].

Next, to find a_i, we only need to equate the coefficient of θ_i in the LHS and RHS of Eq. (36), that implies (note that $\beta > 0$)

$$\begin{aligned} &\sum_{j\neq i} \kappa_{i,j} a_j + 2a_i = (1+\delta) + \sum_{j\neq i} \kappa_{i,j} \equiv v_i \\ \Rightarrow \quad & A\boldsymbol{a} = \boldsymbol{v}, \end{aligned} \tag{37}$$

where $A \equiv \frac{1}{\sigma^2}\Sigma + I$, and I denotes the identity matrix. Since A is positive define, it is invertible and thus

$$\boldsymbol{a} = A^{-1}\boldsymbol{v}, \tag{38}$$

that is

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} 2 & \kappa_{1,2} & \cdots & \kappa_{1,n} \\ \kappa_{2,1} & 2 & \cdots & \kappa_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{n,1} & \kappa_{n,2} & \cdots & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1+\delta+\sum_{j\neq 1}\kappa_{1,j} \\ 1+\delta+\sum_{j\neq 2}\kappa_{2,j} \\ \vdots \\ 1+\delta+\sum_{j\neq n}\kappa_{n,j} \end{pmatrix}.$$

Thus,

$$\begin{aligned} \boldsymbol{a} = \left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1} \left(\delta\mathbf{1} + \frac{1}{\sigma^2}\Sigma\mathbf{1}\right) &= \left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1} \left((\delta-1)\mathbf{1} + \mathbf{1} + \frac{1}{\sigma^2}\Sigma\mathbf{1}\right) \\ &= \mathbf{1} + (\delta-1)\left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1}. \end{aligned}$$

Similarly, we derive b_i. Equating the terms that are independent of θ_i in the LHS and RHS of Eq. (36) implies

$$\begin{aligned} &\alpha - \gamma - \beta\left(\sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right) = 2\beta b_i \\ \Rightarrow \quad & \alpha - \gamma - \beta\left(2b_i + \sum_{j\neq i} b_j + R - q_i^f + \delta R/n\right) = 0. \end{aligned} \tag{39}$$

Let $b \equiv \sum_{j=1}^n b_j$. Taking a sum over all i from Eq. (39) gives

$$b = \frac{n\alpha - n\gamma - n\beta R - \beta(-Q^f + \delta R)}{(n+1)\beta} \tag{40}$$

where $Q^f \equiv \sum_{i=1}^n q_i^f$. In addition, from Eq. (39) we have

$$\alpha - \beta b - \beta R - \beta(-q_i^f + \delta R/n) - \gamma = \beta b_i.$$

The LHS of the above equation can be simplified as follows:

$$\begin{aligned}
\text{LHS} &= (\alpha - \gamma) - \beta R - \beta(q_i^f + \delta R/n) - \beta b \\
&= \frac{1}{n+1}\Big((n+1)[(\alpha-\gamma) - \beta R - \beta(-q_i^f + \delta R/n)] - n(\alpha-\gamma) + n\beta R + \beta(-Q^f + \delta R)\Big) \\
&= \frac{1}{n+1}\left((\alpha-\gamma) - \beta R - \beta\left[\left(-(n+1)q_i^f + \frac{n+1}{n}\delta R\right) + Q^f - \delta R\right]\right) \\
&= \frac{1}{n+1}\left((\alpha-\gamma) - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\right)
\end{aligned}$$

Therefore, we finally have

$$b_i = \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta\left[Q^f - (n+1)q_i^f + R + \delta R/n\right]\right). \tag{41}$$

Equations Eq. (41) and Eq. (38) summarize the unique linear equilibrium in the production stage.

Stage 1: Contracting stage Equipped with the results from the production stage, we next evaluate the amount of optimal forward contract q_i^f for each generator $i = 1, 2, \cdots, n$, at the average market price $p_i^f = E_\theta[p]$ (due to the no arbitrage assumption). This is achieved by computing expected payoff of each generator. Thus, generator i's optimal choice for q_i^f should maximize

$$E_\theta\left[\left(\alpha - \beta(q_i(q_i^f, q_{-i}^f) + R_i + \sum_{j\neq i} R_j + \sum_{j\neq i} q_j(q_j^f, q_{-j}^f))\right)(q_i(q_i^f, q_{-i}^f) + \delta R_i) - \gamma q_i(q_i^f, q_{-i}^f)\right].$$

The optimal choice in the production stage is linear in the observed information and is in the form of $q_i(\theta_i) = b_i - a_i\theta_i$. Characterization of a_i and b_i (see Eq. (41) and Eq. (38)) gives

$$\frac{\partial q_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f}.$$

By the above equality and the fact that $E[\theta_i] = 0$ and $E[R_i] = R/n$, it is sufficient to find q_i^f maximizing

$$(\alpha - \beta(b(q_i^f, q_{-i}^f) + R))(b_i(q_i^f, q_{-i}^f) + \delta R/n) - \gamma b_i(q_i^f, q_{-i}^f).$$

The FOC gives

$$(\alpha - \beta(b(q_i^f, q_{-i}^f) + R))\frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f} - \beta \frac{\partial b(q_i^f, q_{-i}^f)}{\partial q_i^f}\left(b_i(q_i^f, q_{-i}^f) + \delta R/n\right) - \gamma \frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = 0 \tag{42}$$

Since $\frac{\partial b_i(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{n}{n+1}$ (see Eq. (41)) and $\frac{\partial b(q_i^f, q_{-i}^f)}{\partial q_i^f} = \frac{1}{n+1}$ (see Eq. (40)), thus Eq. (42) gives

$$(\alpha - \beta(b(q_i^f, q_{-i}^f) + R))\frac{n}{n+1} - \gamma \frac{n}{n+1} - (b_i(q_i^f, q_{-i}^f) + \delta R/n)\frac{\beta}{n+1} = 0. \tag{43}$$

Multiplying Eq. (43) in $(n+1)$ and rearranging terms yield

$$-\beta(b_i + nb(q_i^f, q_{-i}^f)) + n(\alpha - \gamma) - \beta(\delta R/n + nR) = 0.$$

By symmetry $b_1(q_1^f, q_{-1}^f) = b_2(q_2^f, q_{-2}^f) = \cdots = b_n(q_n^f, q_{-n}^f)$, thus

$$-\beta(n^2+1)b_i(q_i^f, q_{-i}^f) + n(\alpha - \gamma) - \beta(\delta R/n + nR) = 0. \tag{44}$$

Further, Eq. (41) implies that (note that by symmetry $q_1^f = q_2^f = \cdots = q_n^f$)

$$-\beta(n^2+1)b_i(q_i^f, q_{-i}^f) = -\frac{n^2+1}{n+1}\left(\alpha - \gamma - \beta\left[-q_i^f + R + \delta R/n\right]\right)$$

Plugging this into Eq. (44) gives

$$\begin{aligned} &- (n^2+1)\left[\alpha - \gamma - \beta(-q_i^f + R + \delta R/n)\right] + \\ &\quad (n^2+n)(\alpha - \gamma) - (n+1)\beta(\delta R/n + nR) = 0. \end{aligned} \tag{45}$$

Rearranging terms implies

$$\begin{aligned} (n^2+1)\beta q_i^f &= (n-1)(\alpha - \gamma) + (n^2+1)\beta(R + \delta R/n) - (n+1)\beta(\delta R/n + nR) \\ &= (n-1)(\alpha - \gamma) + \beta((n^2+1)R - (n^2+n)R + [(n^2+1) - (n+1)]\delta R/n) \\ &= (n-1)(\alpha - \gamma) + \beta(-(n-1)R + (n-1)\delta R) \\ &= (n-1)[\alpha - \gamma + \beta(-R + \delta R)]. \end{aligned}$$

Thus

$$q_i^f = \frac{n-1}{(n^2+1)\beta}\Big(\alpha - \gamma + \beta(-R+\delta R)\Big). \tag{46}$$

Finally, we next find b_i. Plugging q_i^f into Eq. (41) gives

$$\begin{aligned}
b_i &= \frac{1}{(n+1)\beta}\Big(\alpha - \gamma - \beta(-q_i^f + R + \delta R/n)\Big) \\
&= \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \beta\left[-\frac{n-1}{(n^2+1)\beta}\Big(\alpha-\gamma+\beta(-R+\delta R)\Big) + R + \delta R/n\right]\right) \\
&= \frac{1}{(n+1)\beta}\left(\alpha - \gamma - \left[\frac{-(n-1)[\alpha-\gamma+\beta(-R+\delta R)] + (n^2+1)\beta(R+\delta R/n)}{n^2+1}\right]\right) \\
&= \frac{1}{(n+1)\beta}\left(\frac{(n^2+n)(\alpha-\gamma) + \beta\left(-(n^2+n)R - \frac{n^2+n}{n^2}\delta R\right)}{n^2+1}\right) \\
&= \frac{n}{(n^2+1)\beta}\left(\alpha + \beta\left(-R - \frac{\delta R}{n^2}\right) - \gamma\right).
\end{aligned}$$

With the characterization of b_i, $i = 1, 2\cdots, n$, the proof is complete. ∎

E Derivations of price volatility and welfare/profit

E.1 Price volatility

Proof of Proposition 1 and Lemma 6. As shown in Proposition 5 the optimal production strategy is in form of $q_i(R_i) = b_i - a_i\theta_i$, where b_i and a_i are scalars. Thus

$$\begin{aligned}
Var(p) &= Var\left(\alpha - \beta\left(\sum_{i=1}^{n}(b_i - a_i\theta_i) + \sum_{i=1}^{n}(R/n + \theta_i)\right)\right) \\
&= \beta^2 Var\left(\sum_{i=1}^{n}(a_i - 1)\theta_i\right) \\
&= \beta^2\left(\boldsymbol{a}^T\Sigma\boldsymbol{a} - 2\boldsymbol{a}^T\Sigma\mathbf{1} + \mathbf{1}^T\Sigma\mathbf{1}\right) \\
&= \beta^2(\boldsymbol{a}-\mathbf{1})^T\Sigma(\boldsymbol{a}-\mathbf{1}) \\
&= \beta^2(\delta-1)^2\ \mathbf{1}^T\left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\Sigma\left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1}\mathbf{1} \\
&= \beta^2(\delta-1)^2\ \mathbf{b}^T\Sigma\mathbf{b},
\end{aligned}$$

where $\mathbf{b} \equiv \left(\mathbf{I} + \frac{1}{\sigma^2}\Sigma\right)^{-1} \mathbf{1}$.

We next derive price volatility when Definition 2 holds. First note that for regular configurations, we have $v_i = 1 + \delta + \sum_{j \neq i} \kappa_{i,j} = 1 + \delta + K$, for all $i = 1, 2, \cdots, n$. Therefore, due to Eq. (37), the optimal a_i satisfies

$$2a_i + \sum_{j \neq i} a_j \kappa_{i,j} = v_i = 1 + \delta + K.$$

Thus, by symmetry, we have $a_1 = \cdots = a_n = \frac{1+\delta+K}{2+K}$. Consequently,

$$Var(p) = \beta^2 \left(1 - \frac{1+\delta+K}{2+K}\right)^2 \mathbf{1}^T \Sigma \mathbf{1} = n\sigma^2\beta^2 \left(\frac{1-\delta}{2+K}\right)^2 (1+K),$$

completing the proof. ∎

Proof of Proposition 2. Since the decay factor $\zeta \in (0\ 1)$, thus

$$K_{\text{cycle}} = 2\left(\zeta + \zeta^2 + \cdots + \zeta^{\frac{n-1}{2}}\right) = 2\zeta \left(\frac{1 - \zeta^{\frac{n-1}{2}}}{1 - \zeta}\right) \leq (n-1)\zeta = K_{\text{complete}}$$

Moreover, by the characterization of price volatility in Lemma 6, it is clear that $\frac{\partial Var(p)}{\partial K} \leq 0$. Thus, price volatility is in decreasing the decay factor because

$$\frac{\partial Var(p)}{\partial \zeta} = \frac{\partial Var(p)}{\partial K} \underbrace{\frac{\partial K}{\partial \zeta}}_{\geq 0} \leq 0.$$

This implies that price volatility in the complete model is less than the cycle model, completing the proof. Note that since $\kappa_{i,j} \in \{0, \kappa\}$, for any $i \neq j$, thus given the definition of regular structures, for any regular configuration with n renewable plants, $K^{(n)}_{\text{cycle}} < K^{(n)}_{\text{regular}} < K^{(n)}_{\text{complete}}$. ∎

E.2 Welfare/Profit analysis

Before proving Theorem 3, we present two intuitive results about the impact of R and δ on each thermal producer's profit. We show diversified energy portfolios are always beneficial for thermal producers. However, the profit consequences of increasing renewables for diversified thermal producers (i.e. $\delta > 0$) crucially depends on δ. That is, depending on the extent of δ, increasing renewable supply can be beneficial or detrimental for thermal producers. In fact, our model suggests there exists a *unique* threshold δ^* for which when thermal producers have a low share from renewable outcome (i.e. $\delta < \delta^*$) their benefit decreases with increasing renewable supply, but when their share is sufficiently high (i.e. $\delta > \delta^*$), increasing renewable supply is actually beneficial for them.

Proposition 3. *The payoff of each non-diversified thermal producer monotonically decreases with increasing renewable supply on the grid, i.e.* $\frac{\partial \Pi_i}{\partial R} < 0$ *if* $\delta = 0$. *However, the payoff of each thermal producer always arises via diversification, i.e.* $\frac{\partial \Pi_i}{\partial \delta} > 0$.

Proof of Proposition 3. Due to Theorem 2, $\frac{\partial p}{\partial \delta} = \frac{\beta R}{n+1} > 0$, and $\frac{\partial q_i}{\partial \delta} = \frac{-\beta R/n}{(n+1)\beta}$ thus $\frac{\partial q_i}{\partial \delta} + \frac{R}{n} = \frac{R}{n}(1 - \frac{1}{n+1}) > 0$. Moreover,

$$\frac{\partial \Pi_i}{\partial \delta} = \underbrace{\frac{\partial p}{\partial \delta}}_{\geq 0} \left(q_i + \frac{\delta R}{n} \right) + p \underbrace{\left(\frac{\partial q_i}{\partial \delta} + \frac{R}{n} \right)}_{\geq 0} \underbrace{-\gamma \frac{\partial q_i}{\partial \delta}}_{\geq 0} \geq 0.$$

Next, assume $\delta = 0$. Thus, $\frac{\partial p}{\partial R} = \frac{-\beta}{n+1} < 0$, and $\frac{\partial q_i}{\partial R} = \frac{-1}{(n+1)} < 0$. Moreover,

$$\begin{aligned} \frac{\partial \Pi_i}{\partial R} &= \frac{\partial p}{\partial R} q_i + p \frac{\partial q_i}{\partial R} - \gamma \frac{\partial q_i}{\partial R} \\ &= \frac{-\beta}{n+1} q_i + (p - \gamma) \frac{-1}{n+1} < 0, \end{aligned}$$

where the last inequality holds because $p - \gamma > 0$. ∎

Proposition 4. *There exists a unique* $\delta^* \in (0, 1)$ *such that if* $\delta < \delta^*$ *then with increasing renewable supply the profit of each diversified thermal producer decreases, i.e.* $\frac{\partial \Pi_i}{\partial R} < 0$. *However, if* $\delta > \delta^*$ *then each diversified thermal producer will be better off with increasing renewable supply, i.e.* $\frac{\partial \Pi_i}{\partial R} > 0$.

Proof of Propostion 4. Due to symmetry, the profit of each thermal producer at the equilibrium can be written as $\Pi_i = (q_i + \delta R/n)p - \gamma q_i = \frac{1}{n}[(Q + \delta R)p - \gamma Q]$, recall that $Q = \sum_{i=1}^{n} q_i = n q_i$ (at the equilibrium). Further, due to Theorem 2, $p = \frac{1}{(n+1)}(\alpha + \beta(-R + \delta R) + n\gamma)$ and $Q = \frac{n}{(n+1)\beta}(\alpha - \gamma - \beta(R + \delta R/n))$. Plugging p and Q into the profit of the thermal producer i gives

$$
\begin{aligned}
\Pi_i &= \frac{1}{n}\left[(Q+\delta R)p - \gamma Q\right] \\
&= \frac{1}{\beta n(n+1)}\left[\frac{1}{n+1}\Big(\alpha+\beta(-R+\delta R)+n\gamma\Big)\Big(n(\alpha-\gamma)-\beta(nR+\delta R)+\beta(n+1)\delta R\Big)\right. \\
&\qquad \left. -\gamma n\Big(\alpha-\gamma-\beta(R+\delta R/n)\Big)\right]
\end{aligned}
$$

Let us define $\Lambda \equiv [...]$ (note that the roots of $\frac{\partial \Pi_i}{\partial R}$ and $\frac{\partial \Lambda}{\partial R}$ are equivalent). Thus,

$$
\begin{aligned}
\frac{\partial \Lambda}{\partial R} &= \frac{\beta(\delta-1)}{n+1}\Big(n(\alpha-\gamma)-\beta(nR+\delta R)+\beta(n+1)\delta R\Big) \\
&\quad + \frac{-\beta(\delta+n)+(n+1)\delta\beta}{n+1}\Big(\alpha+\beta(-R+\delta R)+n\gamma\Big) \\
&\quad + \gamma n\beta\left(1+\frac{\delta}{n}\right) \\
&\equiv f(\delta).
\end{aligned}
$$

It is easy to see that $f(\delta)$ is quadratic in δ, thus it looks like $f(\delta) = x\delta^2 + y\delta + z$ (where x, y, and z are all independent of δ). First note that $x = \frac{2\beta^2 Rn}{n+1} > 0$, thus $f(\delta)$ is convex. Next we show $f(\delta)$ has a unique zero lying between zero and one. To achieve this we show $f(0) < 0$ and $f(1) > 0$ (see the following Figure) since $f(\delta)$ is continuous and quadratic thus has unique zero in $(0, 1)$.

$$
\begin{aligned}
f(0) &= -\beta\left(\frac{n(\alpha-\gamma)-\beta nR}{n+1}\right) - \frac{n\beta}{n+1}(\alpha-\beta R+n\gamma)+\gamma n\beta \\
&= \frac{-2n\beta}{n+1}(\alpha-\gamma-\beta R) \\
&< 0
\end{aligned}
$$

where the last inequality holds since $\alpha - \gamma - \beta R > 0$.

$$
\begin{aligned}
f(1) &= \gamma n\beta\left(1+\frac{1}{n}\right) \\
&> 0.
\end{aligned}
$$

Since $f(1) > 0$ and $f(0) < 0$, thus there exists a unique $\delta^* \in (0, 1)$ for which $f(\delta^*) = 0$. As a result, $\frac{\partial \Pi_i}{\partial R}|_{\delta=\delta^*} = 0$, and consequently, $\frac{\partial \Pi_i}{\partial R}|_{\delta<\delta^*} < 0$ and $\frac{\partial \Pi_i}{\partial R}|_{\delta>\delta^*} > 0$, completing

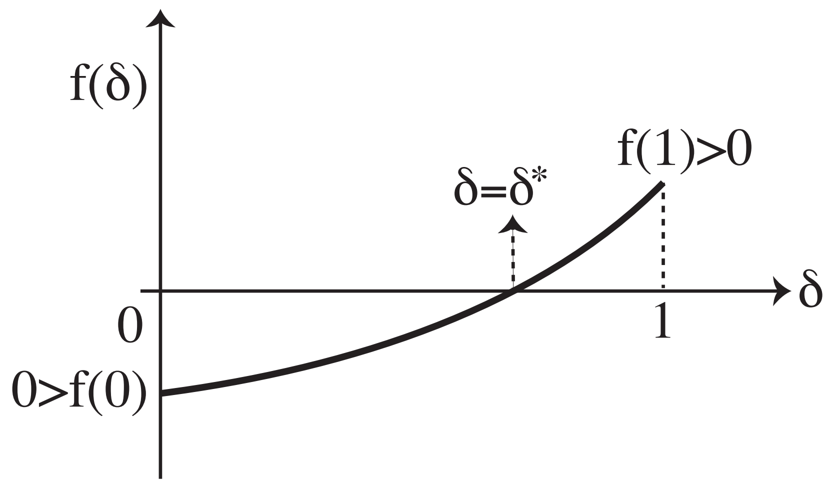

Figure 6: There exists a unique $\delta^* \in (0,1)$ for which the behavior of Π_i with respect to R switches.

the proof.

∎

Proof of Theorem 3. As discussed when thermal producers are competitive $\mathcal{W}(CE) = \frac{(\alpha-\gamma)^2}{2\beta} + \gamma R$, that is independent of δ. Next, we consider the case in which thermal producers are strategic. Thus, at the corresponding Nash equilibrium the welfare is given by

$$\begin{aligned}\mathcal{W}(NE) &= (Q_{NE} + \delta R)p_{NE} - \gamma Q_{NE} + p_{NE}(1-\delta)R + \frac{(\alpha - p_{NE})^2}{2\beta} \\ &= (p_{NE} - \gamma)Q_{NE} + Rp_{NE} + \frac{(\alpha - p_{NE})^2}{2\beta}\end{aligned}$$

where (as shown in Theorem 2) the overall production is given by $Q_{NE} = \frac{n}{(n+1)\beta}(\alpha - \gamma - \beta(R + \delta R/n))$ and the resulting spot price satisfies $p_{NE} = \frac{1}{n+1}(\alpha + \beta(-R + \delta R) + n\gamma)$. Therefore

$$\begin{aligned}\frac{\partial \mathcal{W}(NE)}{\partial \delta} &= \left(\frac{\partial p_{NE}}{\partial \delta}\right) Q_{NE} + (p_{NE} - \gamma)\frac{\partial Q_{NE}}{\partial \delta} + \frac{\partial p_{NE}}{\partial \delta} R - \frac{1}{\beta}(\alpha - p_{NE})\frac{\partial p_{NE}}{\partial \delta} \\ &= \underbrace{(p_{NE} - \gamma)}_{>0} \underbrace{\frac{\partial Q_{NE}}{\partial \delta}}_{<0} \\ &< 0, \end{aligned} \tag{47}$$

note that $p_{NE} > \gamma$, because $\alpha + \beta(-R + \delta R) - \gamma > 0$. Equation Eq. (47) immediately implies $\frac{\partial}{\partial \delta}\left(\frac{\mathcal{W}(CE)}{\mathcal{W}(NE)}\right) > 0$, completing the proof. ∎

F Price volatility: General Spatial Configurations

In this section through examples we aim to consider the effect of general correlation structures on the price volatility. Importantly, we show comparison of price volatility among different network structures crucially depends on the way that the underlying structures are normalized. To this end, we consider three normalizations as follows.

F.1 Normalization 1: Fixed distance between any neighboring plants

We consider path, cycle, barbell and complete network structures, depicted in the following figure.

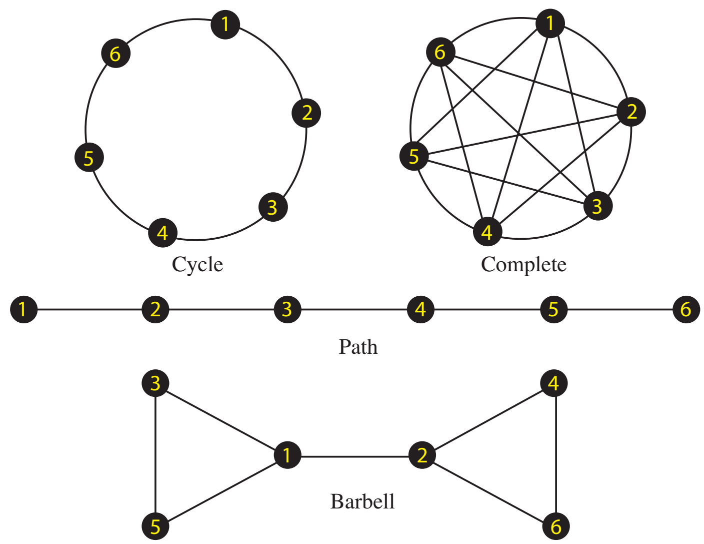

Figure 7: Cycle, Complete, Path and Barbell network structures.

We assume the distance between any two immediate neighbor plants is normalized to 1, and in any network $d(\ell_i, \ell_j)$ is the shortest distance between plants i and j within the network (thus $d(\ell_i, \ell_i) = 0$ for all i). We assume $\sigma^2 = 1$. To capture that correlation in renewable supply at any two plants, i.e. $\kappa_{i,j}$, decays with their distance we assume $\kappa_{i,j} = \zeta^{d(\ell_i, \ell_j)}$ where $0 < \zeta < 1$ is the decay factor and $d(\ell_i, \ell_j)$ is the shortest distance between the plants. For example, the variance-covariance matrix of the above Barbell and Path networks are as follows:

$$\Sigma_{\text{Barbell}} = \begin{pmatrix} 1 & \zeta & \zeta & \zeta^2 & \zeta & \zeta^2 \\ \zeta & 1 & \zeta^2 & \zeta & \zeta^2 & \zeta \\ \zeta & \zeta^2 & 1 & \zeta^3 & \zeta & \zeta^3 \\ \zeta^2 & \zeta & \zeta^3 & 1 & \zeta^3 & \zeta \\ \zeta & \zeta^2 & \zeta & \zeta^3 & 1 & \zeta^3 \\ \zeta^2 & \zeta & \zeta^3 & \zeta & \zeta^3 & 1 \end{pmatrix}, \quad \Sigma_{\text{Path}} = \begin{pmatrix} 1 & \zeta & \zeta^2 & \zeta^3 & \zeta^4 & \zeta^5 \\ \zeta & 1 & \zeta & \zeta^2 & \zeta^3 & \zeta^4 \\ \zeta^2 & \zeta & 1 & \zeta & \zeta^2 & \zeta^3 \\ \zeta^3 & \zeta^2 & \zeta & 1 & \zeta & \zeta^2 \\ \zeta^4 & \zeta^3 & \zeta^2 & \zeta & 1 & \zeta \\ \zeta^5 & \zeta^4 & \zeta^3 & \zeta^2 & \zeta & 1 \end{pmatrix}.$$

In this section we assume $\beta = 1$. Applying Proposition 1 we can characterize the price volatility for the above configurations. Figure 9 visualizes the price volatility of these networks when the decay factor varies from 0 to 1, with different share for thermal producers from renewable supply, i.e. δ. Consistent with Proposition 1 price volatility decreases with increasing δ. In addition price volatility decreases with increasing the decay factor. This is because decay factor inversely related to the distance between the plants. Thus, high decay factor means low distance among the plants, decreasing the price volatility which is due to the lower misscoordinations among (close) competitors. In addition, with changing the decay factor and δ, price volatility in these networks uniformly follows a pattern that $Var_{\text{Path}}(p) \geq Var_{\text{Cycle}}(p) \geq Var_{\text{Barbell}}(p) \geq Var_{\text{Complete}}(p)$.

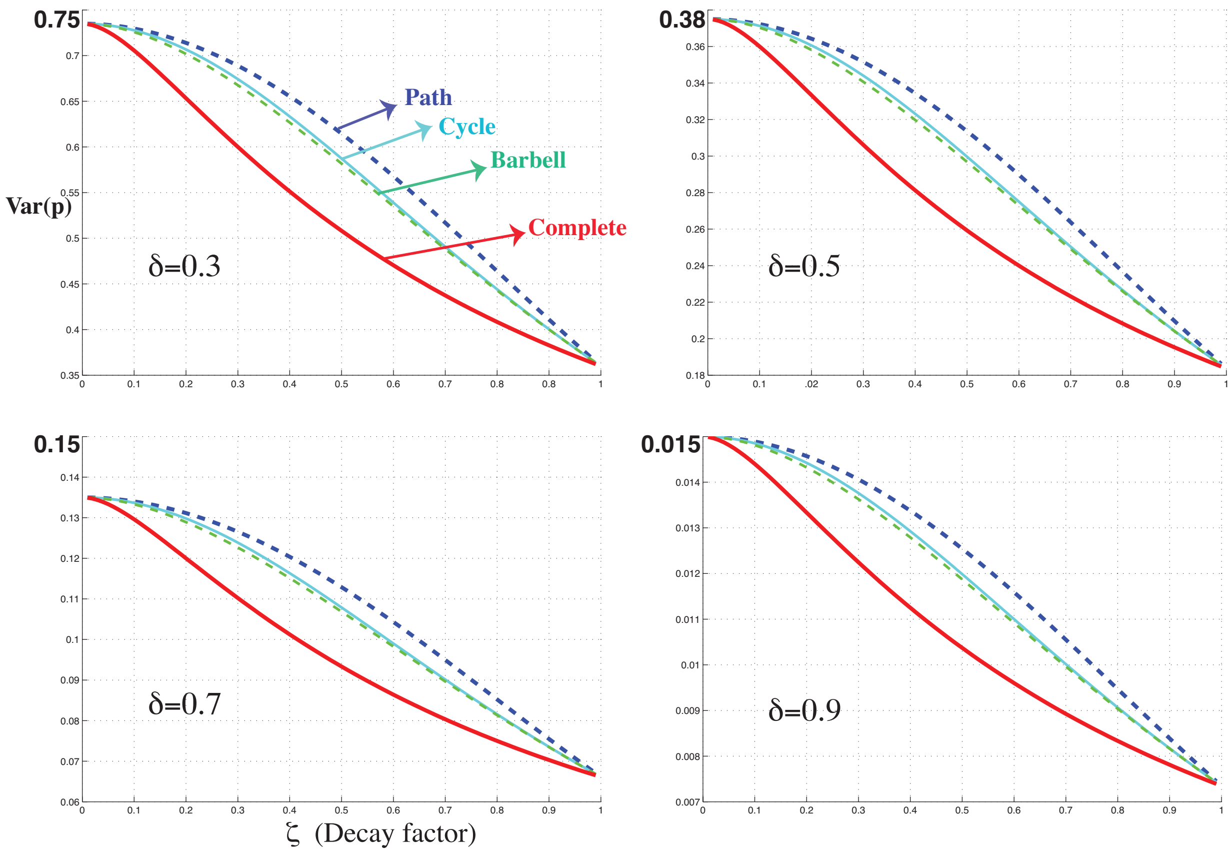

Figure 8: Price volatility in Cycle, Complete, Path and Barbell networks with respect to the decay factor $\zeta \in (0,1)$ when thermal producers own some share from renewable outcome, i.e. $\delta = .3, .5, .7, .9$. *Price volatility in the Path (Complete) structure is uniformly higher (lower) than the others.*

F.2 Normalization 2: The same distance for the farthest plants

For this normalization we compare Path and Barbell structures, given that the distance for their farthest plants is the same. In the Path network the correlation between any two "neighboring" plants is ζ, where $\zeta \in (0, 1)$ is the decay factor. However, the Barbell network consists of two cliques located far from each other. In each clique the correlation between any two neighbor plants is ζ. But, since by this normalization, in the Path and Barbell networks the distance between the two farthest plants should be the same, thus the correlation between the two farthest plants in these networks is $\zeta^{\text{maximum distance}} = \zeta^{2n-1}$ (see Figure 10).

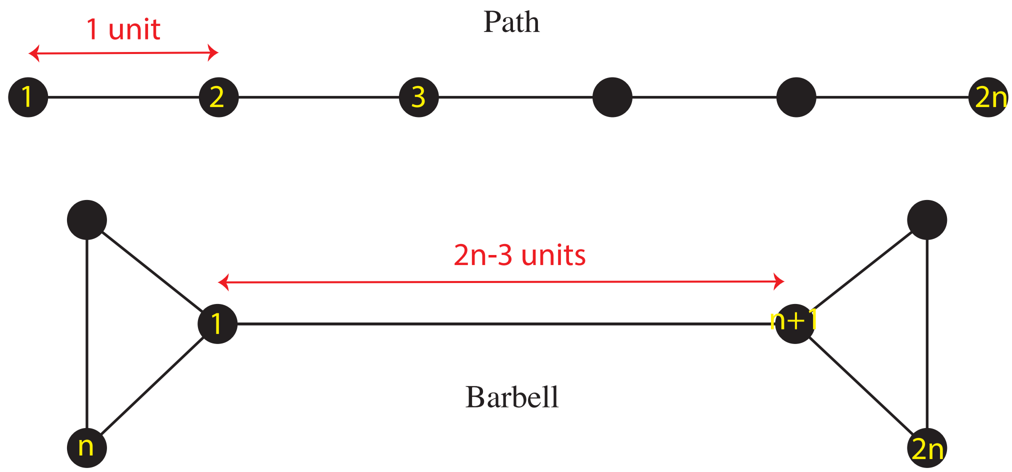

Figure 9: Path and Barbell networks. The same distance for the farthest plants.

Now, suppose the decay factor changes from zero to one. Then, as shown in Figures 11 and 12, in contrast to the previous case, price volatility with this normalization does not change uniformly in these network structures. In fact, when decay factor is small (i.e. low correlation in renewable supply for neighboring plants) price volatility in Path is higher than Barbel structures. But, when decay factor is sufficiently high (i.e. high correlation in renewable supply for neighboring plants) then price volatility in the Path structure is lower than the Barbell.

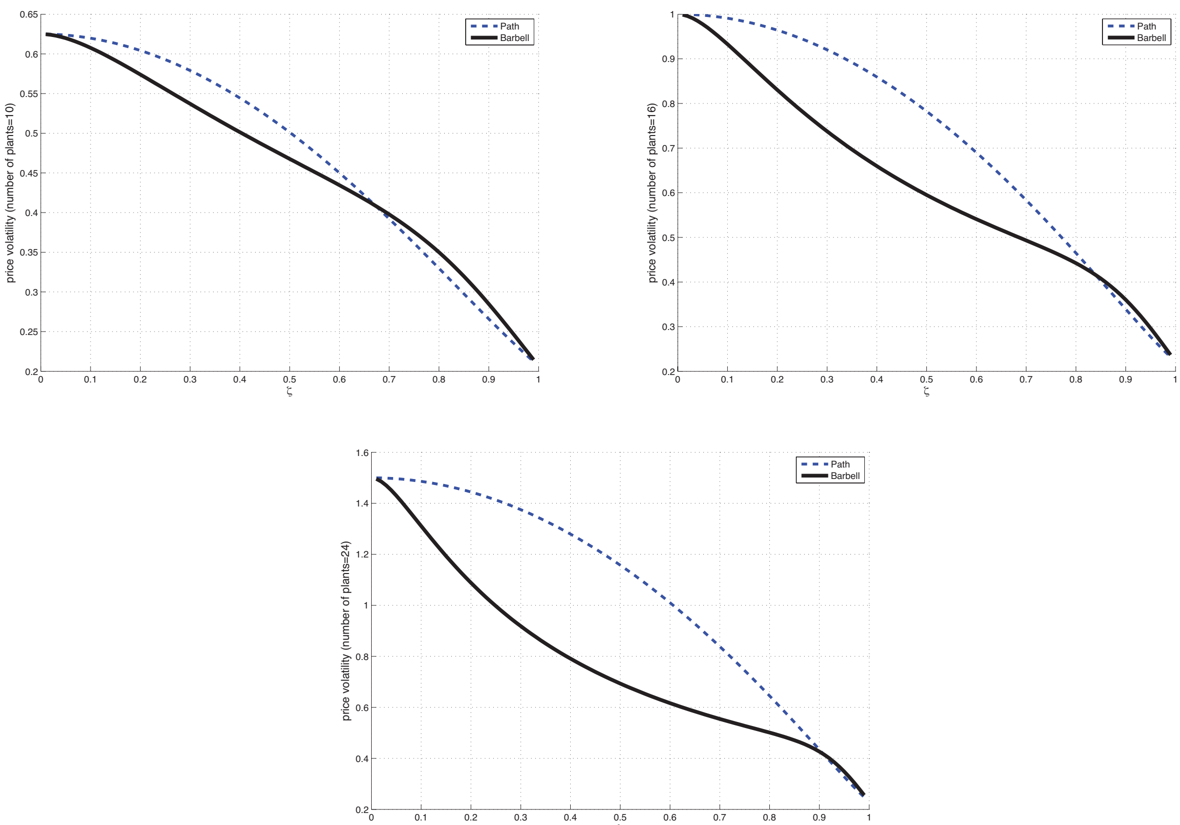

Figure 10: Barbell vs Path networks (Effect of distance). When nodes are close (i.e. high ζ), the price volatility in the path network is lower. However when nodes are far (i.e. low ζ), the price volatility in the barbell network is lower.

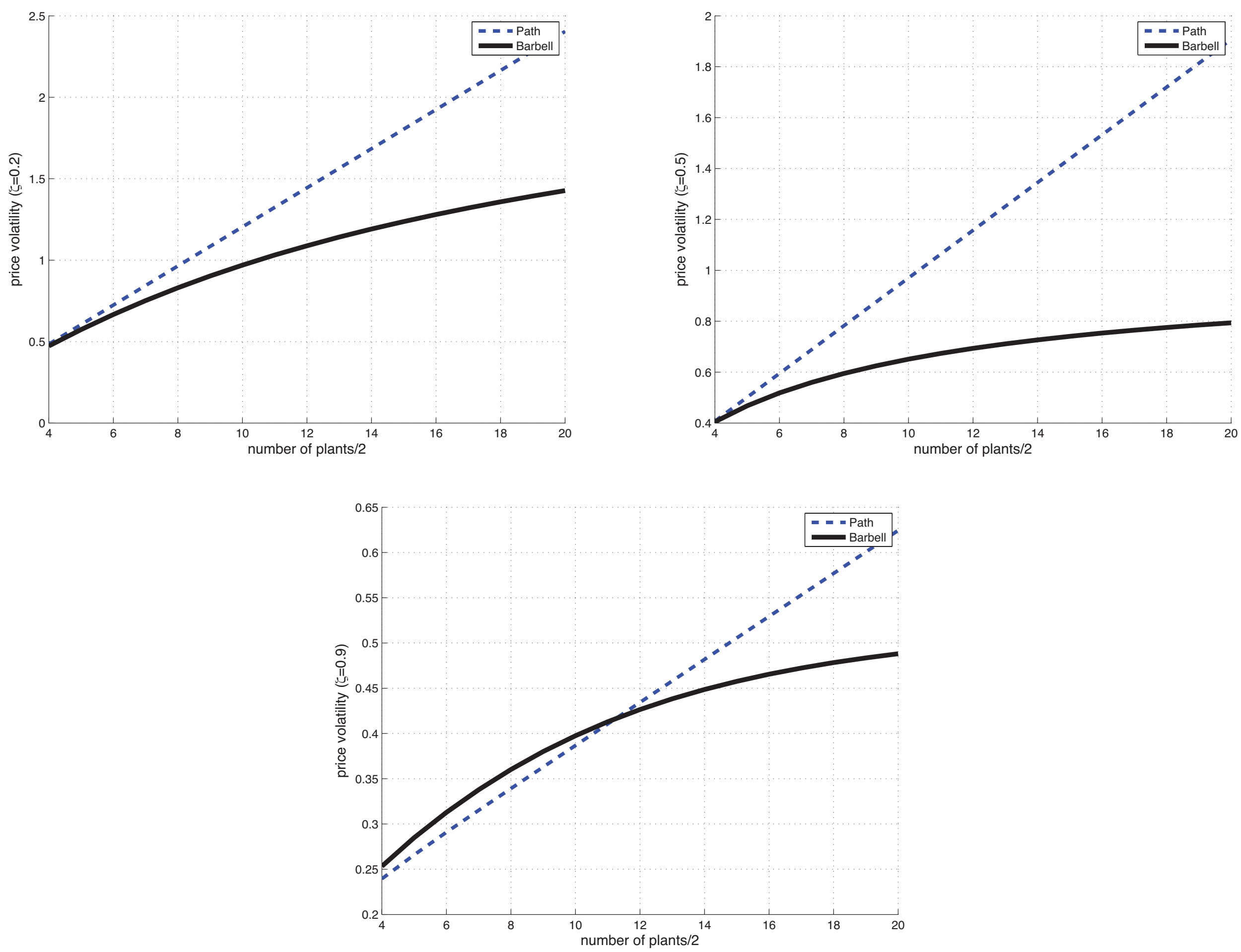

Figure 11: Barbell vs Path networks (Effect of size). The same intuition as in the above.

F.3 Normalization 3: The same total correlation

In this normalization we assume the total correlation in the underlying structures is the *same*, i.e. $\mathbf{1}^T \Sigma_{structure} \mathbf{1} = \text{fixed}$.

The construction is as follows. Let us start with a path network with $2n$ nodes. In the path network the corresponding variance-covariance matrix (i.e. Σ_{Path}) is such that (for a given decay factor $\zeta \in (0,1)$) the $Cov(\theta_i, \theta_j) = \zeta^{d_{ij}}$, where d_{ij} denotes the length of the shortest path from i to j on the path.

$$\Sigma_{\text{Path}} = \begin{pmatrix} 1 & \zeta & \zeta^2 & \cdots & \zeta^{2n-1} \\ \zeta & 1 & \zeta & \cdots & \zeta^{2n-2} \\ \zeta^2 & \zeta & 1 & \cdots & \zeta^{2n-3} \\ \vdots & & & \ddots & \vdots \\ \zeta^{2n-1} & \zeta^{2n-2} & \zeta^{2n-3} & \cdots & 1 \end{pmatrix}$$

Next moving to the barbell and the complete networks, the constructions of their variance-covariance matrices need to satisfy $\mathbf{1}^T \Sigma_{Complete} \mathbf{1} = \mathbf{1}^T \Sigma_{Path} \mathbf{1} = \mathbf{1}^T \Sigma_{Barbell} \mathbf{1}$ (resulting in normalization in the total correlation). As a result, for the Barbell structure that consists of two cliques (each with n nodes), the $Cov(\theta_i, \theta_j) = q_b$ if $i \neq j$ and both belongs to a same clique, however, when i and j are in different cliques then $Cov(\theta_i, \theta_j) = \alpha q_b$ where $\alpha \in (0,1)$ and $q_b \equiv \frac{\mathbf{1}^T \Sigma_{Path} \mathbf{1} - 2n}{2n((\alpha+1)n-1)}$. It can be easily shown that with this q_b, $\mathbf{1}^T \Sigma_{Path} \mathbf{1}$ and $\mathbf{1}^T \Sigma_{Barbell} \mathbf{1}$ will be equal. Following the same argument, for the complete network we need to choose $Cov(\theta_i, \theta_j) = q_c$ where $q_c \equiv \frac{\mathbf{1}^T \Sigma_{Path} \mathbf{1} - 2n}{2n(2n-1)}$ (for all $i \neq j$), so as to have $\mathbf{1}^T \Sigma_{Complete} \mathbf{1} = \mathbf{1}^T \Sigma_{Path} \mathbf{1}$.

$$\Sigma_{\text{Complete}} = \begin{pmatrix} 1 & q_c & \cdots & & q_c \\ q_c & 1 & q_c & \cdots & q_c \\ \vdots & & \ddots & & \vdots \\ q_c & q_c & \cdots & 1 & q_c \\ q_c & \cdots & & q_c & 1 \end{pmatrix} = q_c(\mathbf{U}_{2n} - \mathbf{I}_{2n}) + \mathbf{I}_{2n}, \quad q_c = \frac{\mathbf{1}^T \Sigma_{Path} \mathbf{1} - 2n}{2n(2n-1)}$$

$$\Sigma_{\text{Barbell}} = \begin{pmatrix} \mathbf{A}_b & \mathbf{C}_b \\ \mathbf{C}_b & \mathbf{A}_b \end{pmatrix}, \quad q_b = \frac{\mathbf{1}^T \Sigma_{Path} \mathbf{1} - 2n}{2n((\alpha+1)n-1)}, \quad y_b = \alpha x_b \quad (\alpha \text{ is exogenous and lies in} (0,1))$$

where $\mathbf{A}_b = q_b(\mathbf{U}_n - \mathbf{I}_n) + \mathbf{I}_n$ and $\mathbf{C}_b = y_b \mathbf{U}_n$.

With the above constructions we can now compare price volatility in the above structures. Importantly, as is evident from Figure 13 with this normalization the price volatilities in these structures (the red, green and the blue lines/dots) are *all the same*, implying that network structure becomes actually *ineffective*. The solid black line is the price volatility for the barbell network with the previous normalization in which the distance between its two farthest nodes is equal to the distance between the two end nodes of the path network, taken as a way for the normalization.

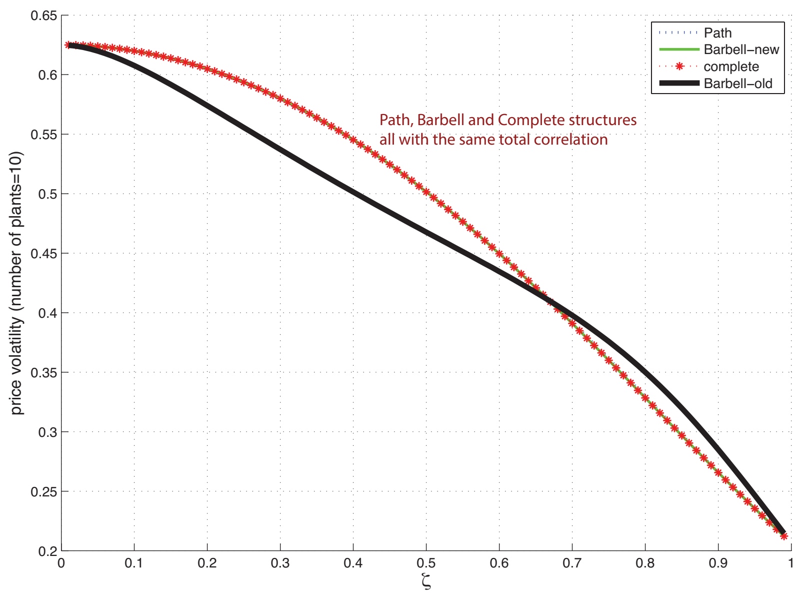

Figure 12: Barbell vs Path vs Complete networks (Network structure becomes ineffective when they all have the same *total correlation*). This figure visualizes the effect of total correlation on the price volatility for different network structures.